Radiation mean free path for non-LTE fully ionized medium*

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Abstract Radiation mean free path (RMFP) is the function of radiation temperature, electron temperature and material density, and needs to apply Planck mean and Rosseland mean, respectively, in the regions of rarefaction wave and shock wave. The analytical formulae for the two kinds of RMFP have been given, by which the RMFP have been calculated for the interesting small atomic number Z. The calculated results show that the RMFP in non-local-thermal-equilibrium (non-LTE) and in LTE are obviously different and Planck RMFP is 30 times less than Rosseland one.

Keywords Radiation mean free path, Radiation transportation, Non-local thermal equilibrium, Diffusion approximation

1 Introduction

The radiation mean free path (RMFP)[1,2] is a very important physical parameter in calculations of radiation hydrodynamics and transportation. For fully ionized medium the simplified formula of calculating RMFP in LTE has been given in Ref.[3]. However, even though the material is fully ionized in the process of laser indirect driven imploding dynamics, the temperatures of radiation and electrons can not reach a local thermal equilibrium (LTE) because of short time and space scale of laser-targets. The RMFP in non-LTE depends on the temperatures of radiation and electrons, and the medium density. The theory and computational method about RMFP in non-LTE was discussed in Refs.[4,5].

In diffusion approximation the photo energy current can be characterized by Fick law, in which the RMFP contained in thermal conduction coefficiency was adopted Rosseland mean for the thick optic thickness and Planck mean for the thin optic thickness, respectively. In the state with an uniform temperature and density the RMFP obtained by above two different methods are obviously different. The ratio of the two values is about 30. In the process of implosion indirectly driven by laser, the deflagration wave structure is formed by soft X-ray ablation. The state in the region of rarefaction wave is high temperature and low density; but the state in the region of shock wave produced by ablative pressure is low temperature and high density. Therefore, the calculations of radiation energy current need two different kinds of RMFP in the two different regions, respectively. In this paper the analytical formulae of RMFP including Rosseland and Planck means for the special case of fully ionized medium are given, which is convenient to numerical calculation and physical analysis.

2 Rosseland MFP

In the system with thick optic thickness the formula of RMFP with Rosseland mean, l_R, is

$$l_R = \frac{\int_0^\infty l_\nu \frac{\mathrm{d}B_\nu}{\mathrm{d}T_R}\mathrm{d}\nu}{\int_0^\infty \frac{\mathrm{d}B_\nu}{\mathrm{d}T_R}\mathrm{d}\nu} \tag{1}$$

where l_ν denotes Rosseland MFP of photon with frequency ν, B_ν and T_R are the Planck spectrum intensity and temperature of radiation,

$$B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{(e^{\frac{h\nu}{kT_R}}-1)} \tag{2}$$

$$\frac{\mathrm{d}B_\nu}{\mathrm{d}T_R} = \frac{2k^3T_R^2}{c^2h^2}\frac{u^4e^u}{(e^u-1)^2} \tag{3}$$

where $u \equiv h\nu/(kT_R)$, then

$$\int_0^\infty \frac{\mathrm{d}B_\nu}{\mathrm{d}T_R}\mathrm{d}\nu = \frac{2k^4T_R^3}{c^2h^3}\frac{4\pi^4}{15} \tag{4}$$

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Substituting Eq.(4) into Eq.(1) to give the formula of Rosseland MFP

$$l_R = \frac{15}{4\pi^4}\int_0^\infty \frac{u^4 e^u (e^u-1)^{-2}\mathrm{d}u}{\mu'(u)} \quad (5)$$

where $\mu'(u)$ is effective absorption coefficient, the inverse of which is radiation free path with frequency ν. In the fully ionized medium without process of photoelectric recombination, the absorption coefficient contains only two parts of bremsstrahlung and scattering, that is

$$\mu'(u) = \mu_f(u)(1-e^{-u}) + \mu_s \quad (6)$$

The first term on the right of Eq.(6) is named as effective coefficient of bremsstrahlung absorption, which has the following form:

$$\mu'_f = \mu_f^0 \frac{\sum\alpha_i Z_i \sum\alpha_i Z_i^2}{(\sum\alpha_i M_i)^2}\frac{\rho^2}{T_R^3 T_e^{1/2}}\left(\frac{1-e^u}{u^3}\right) \quad (7)$$

here α_j is the fraction of the element with nuclear charge Z_j and mass number M_j, T_e is electron temperature.

$$\mu_f^0 = \frac{4}{3}\left(\frac{2\pi}{3m_e k}\right)^{1/2}\cdot\frac{h^2 e^6 N_0^2}{k^3 c m_e} = 1.48017$$

The unit of length, time, mass and temperature are μm, 0.1 ns, pg, MK, respectively. The scattering coefficient with Thomson cross sections is

$$\mu_s = N_e\sigma_s = \mu_s^0 \rho \frac{\sum\alpha_i Z_i}{\sum\alpha_i M_i} \quad (8)$$

where $\mu_s^0 = \frac{8\pi}{3}r_0^2 N_\mathrm{A}$, r_0 is the classical electron radius, N_A is the Avogadro number.

The above formulae show that the Rosseland MFP can not still been obtained analytically from Eq.(5) for fully ionized medium. In order to obtain the analytical formula of MFP, it is necessary to make some approximation.

2.1 The scattering plays a main role

The bremsstrahlung absorption can be ignored as $\mu_s \gg \mu_f(u)$, Eq.(5) can be simplified as

$$l_R \approx l_s = \mu_s^{-1} = \frac{2.496\times10^4}{\rho}\frac{\sum\alpha_i M_i}{\sum\alpha_i Z_i}\ [\mu\mathrm{m}] \quad (9)$$

Since $\sum\alpha_i M_i/\sum\alpha_i Z_i \approx 2$ for the most of low and medial elements, a familiar scattering MFP $l_s \approx 5\times10^4/\rho$ (μm) can be obtained from Eq.(9), the MFP in non-LTE is the same as one in LTE, and is inversely proportional to density.

2.2 The bremsstrahlung absorption plays a main role

The scattering is ignored as $\mu_f(u) \gg \mu_s$, Eq.(9) can be simplified as

$$l_R \approx l_f = (\overline{1/m\mu_f})$$

$$= \frac{15}{4\pi^4\mu_f^0}\frac{(\sum\alpha_i M_i)^2}{\sum\alpha_i Z_i\sum\alpha_i Z_i^2}\frac{T_R^3 T_e^{1/2}}{\rho^2}\int_0^\infty\frac{u^7 e^{2u}\mathrm{d}u}{(e^u-1)^2}$$

$$= 132.77\frac{(\sum\alpha_i M_i)^2}{\sum\alpha_i Z_i\sum\alpha_i Z_i^2}\frac{T_R^3 T_e^{1/2}}{\rho^2}[\mu m] \quad (10)$$

The Rosseland MFP is proportional to the three power of radiation temperaure, to the square root of electron temperature and inversely proportional to the square of density. There are obvious differences between non-LTE MFP and LTE MFP. The following formula has been utilized to obtain Eq.(10),

$$\frac{15}{4\pi^4}\int_0^\infty\frac{u^7 e^{2u}}{(e^u-1)^2}\mathrm{d}u = 196.5194$$

2.3 The condition of applying MFP l_f and l_s in the system of non-LTE

2.3.1 The condition of applying MFP, l_s

Introducing parameter $\beta(0< \beta \le 1)$, and defining as following,

$$\beta \equiv \frac{1}{\mu_s}/(\overline{\mu_f'^{-1}}) \quad (11)$$

then the condition of applying scattering MFP can be obtained by means of Eq.(9) and Eq.(10).

$$(T_R T_e^{1/6})_s \ge \left[\frac{1.88\times10^2}{\beta}\left(\sum\alpha_i Z_i^2/\sum\alpha_i M_i\rho\right)\right]^{1/3} \quad (12)$$

The criterion of $(T_R T_e^{1/6})$ increases with increasing density.

2.3.2 The condition of applying bremsstrahlung MFP, l_f

Introducing parameter $\eta (0 < \eta \leq 1)$, and defining as following,

$$\eta \equiv \left(\overline{\mu_f'^{-1}}\right)/\mu_s^{-1} \tag{13}$$

then the condition of applying bremsstrahlung MFP can be obtained by means of Eq.(9) and Eq.(10) as

$$(T_R T_e^{1/6})_s \leq \left(1.88 \times 10^2 \eta \frac{\sum \alpha_i Z_i^2}{\sum \alpha_i M_i} \rho\right)^{1/3} \tag{14}$$

The criterion of $(T_R T_e^{1/6})$, Eq.(14), increases still with increasing density.

It is easy to see from Eqs.(12) and (14) that if taking $\beta = \eta = 0.05$ the formulae are satisfied

$$(T_R T_e^{1/6})_f \leq 0.14 (T_R T_e^{1/6})_s \tag{15}$$

In general, it is appropriate to replace Rosseland MFP with scattering MFP in the case of high temperature, and to apply bremsstrahlung MFP in the case of low temperature when the density and electron temperature of medium are given. But it is necessary to take account of both of them in the state between the above cases. In Fig.1 the divided region of replacing Rosseland MFP wiht l_s and l_f respectively is given on the plane of density and temperature.

It is shown that to replace Rosseland MFP with l_f as the density is less than 1, the product $(T_R T_e^{1/6})$ of temperature is lower than 1.5 MK; and with l_s as density is less than 1, the product $(T_R T_e^{1/6})$ of temperature is higher than 6~17 MK. In the region between two lines Rosseland MFP must be taken account of the contribution from two mechanisms and can be calculated by using an universal interpolated formula given by the following paragraph.

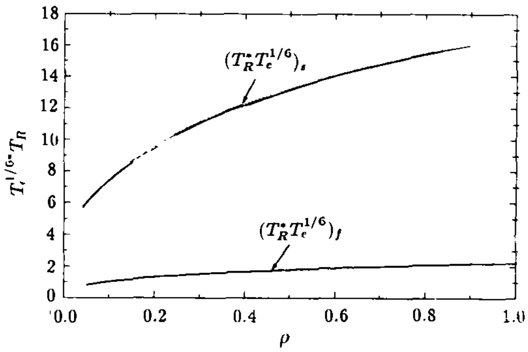

Fig.1 The state region of applying l_s and l_f divided on the plane of temperature and density

2.4 An universal interpolated formula

That Eq.(9) and Eq.(10) are integrated numerically is the universal method in the strict sense. In order to obtain the simplified formula an appropriate method is to replace the numerical integration of Eq.(5) with the inverse interpolation, that is

$$\frac{1}{l_R} = \frac{1}{l_s} + \frac{1}{l_f} \tag{16}$$

The approach formula can be obtained by means of Eq.(9) and Eq.(10).

$$l_R = \left[7.532 \times 10^{-3} \frac{\sum \alpha_i Z_i \sum \alpha_i Z_i^2}{(\sum \alpha_i M_i)^2} \frac{\rho^2}{T_R^3 T_e^{1/2}} + 0.4006 \times 10^{-4} \frac{\rho \sum \alpha_i Z_i}{\sum \alpha_i M_i}\right]^{-1} \tag{17}$$

The results calculated by Eq.(17) are in agreement with one numerically calculated by Eq.(5) within the range of several percents.

3 Planck MFP

In system of thin optical thickness it is used to utilize Planck average:

$$\overline{\mu_P} = \frac{\int_0^\infty \mu_\nu' B_\nu(T_R) \mathrm{d}\nu}{\int_0^\infty B_\nu(T_R) \mathrm{d}\nu} \tag{18}$$

here

$$\int_0^\infty B_\nu(T_R) \mathrm{d}\nu = \frac{2k^4 T_R^4}{c^2 h^3} \int_0^\infty \frac{u^3}{e^u - 1} \mathrm{d}u \tag{19}$$

Substituting Eq.(2) and Eq.(19) into Eq.(18), then

$$\overline{\mu_P} = \frac{15}{\pi^4} \int_0^\infty \mu'(u) \frac{u^3}{e^u - 1} \mathrm{d}u \tag{20}$$

Substituting Eq.(6) and Eq.(7) into Eq.(20), the efficient absorption coefficient of Planck average can be strictly found out

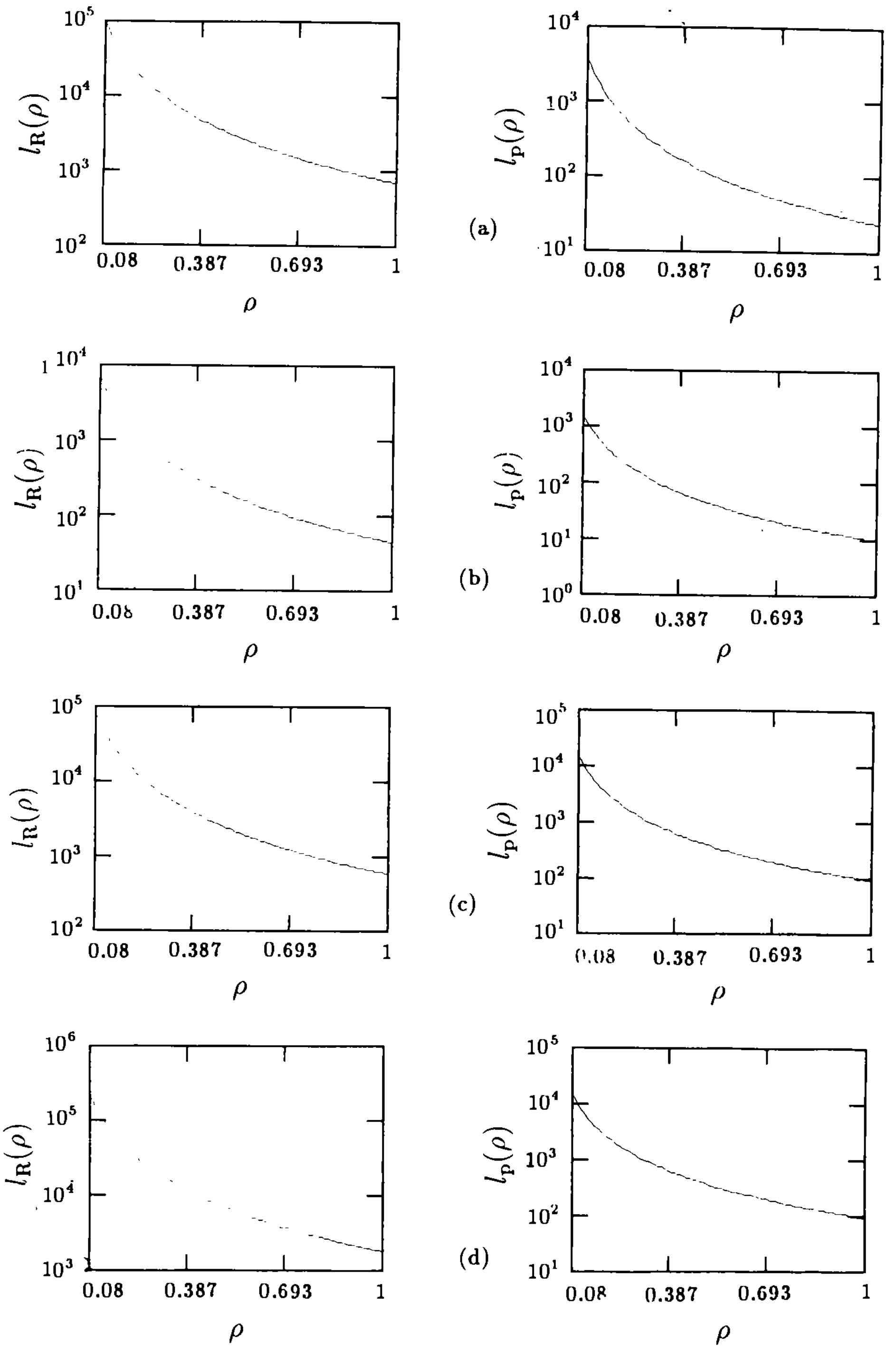

Fig.2 Comparison of Rosseland MFP ($l_R(\rho)$) with Planck MFP ($l_p(\rho)$) in the mediums of ^7LiH (a); C_8H_8 (b); ^4He (c) and DT (d) at $T_R = 1.5$ MK and T_e=1.0 MK

$$\overline{\mu_p} = \overline{\mu_{pf}} + \overline{\mu_s} = 0.228\frac{\sum\alpha_i Z_i \sum\alpha_i Z_i^2}{(\sum\alpha_i M_i)^2}\frac{\rho^2}{T_R^3 T_e^{1/2}} + 0.4006\times10^{-4}\frac{\rho\sum\alpha_i Z_i}{\sum\alpha_i M_i}$$

and the Planck MFP can be obtained from the inverse of above formula

$$l_p = \overline{\mu_p}^{-1} = \left[0.228\frac{\sum\alpha_i Z_i \sum\alpha_i Z_i^2}{(\sum\alpha_i M_i)^2}\frac{\rho^2}{T_R^3 T_e^{1/2}} + 0.4006\times10^{-4}\frac{\rho\sum\alpha_i Z_i}{\sum\alpha_i M_i}\right]^{-1} \tag{21}$$

In comparison of Rosseland and Planck MFP one can find followings. (1) Their weighted function is different, the weighted function of Rosseland average is $\mathrm{d}B_\nu(T_R)/\mathrm{d}T_R$, the peak value of spectrum profile is at u=4, its biggest frequency $h\nu_{\max} = 4kT_R$; while the weighted function of Planck average is $B_\nu(T_R)$, the peak of spectrum profile is at u=2.82, its biggest frequency $h\nu_{\max} = 2.82\,kT_R$. Obviously the spectrum of weighted function of Rosseland average is harder than that of Planck average. So Rosseland MFP is 30 times larger than Planck MFP. (2) The averaging fashion is different, Rosseland MFP is averaged by the inverse of efficient absorption coefficient, and Planck MFP is utilized by the inverse of averaged efficient absorption coefficient. (3) The scattering MFP is independent on the spectrum since the results of two averaging fashions are the same each other.

It is seen from Eq.(17) and Eq.(21) that when bremsstrahlung absorption plays a main role the Rosseland MFP is 30 times larger than Planck MFP for the medium with the same state of temperature and density.

4 Comparison of RMFP in a few mediums

The characteristics of X-ray propagation are studied in all the interesting mediums, such as ^{7}LiH, C_8H_8, ^{4}He and DT, which are in fully ionized state. The RMFP of these mediums have been calculated by simplified Eq.(21). The results are drawn in Fig.2. It is shown that Planck MFP is shorter than Rosseland MFP in the same state of interesting temperature and density, and ^{4}He is the best medium to X-ray propagation; better medium is ^{7}LiH and C_8H_8. Of course, DT is also an advantageous medium to X-ray propagation, but its cost is too high.

5 Discussion

In the process of laser indirect driven imploding dynamics, X-ray transportation is an important topic. It is discussed in the paper that how to apply rationally RMFP on the base of diffusion approximation. It is used to apply Rosseland MFP as optic thickness is thick, and to apply Planck MFP as optic thickness is thin. The RMFP by two kinds of averaging fashions for non-LTE fully ionized medium are given by analytical formulae in this paper. The MFP in non-LTE is different from that in LTE, and is a function of density and temperatures of electrons and radiation. Rosseland MFP is quite different from Planck MFP, the former is 30 times larger than the later in the same state. Some significant results are given in the drawing in this paper, that can be applied to calculations of X-ray transportation in diffusion approximation and to physics analysis.

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