Valence and sea quark mixing in meson states*

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Abstract A meson model with $q\bar{q}$ and $(q\bar{q})^2$ mixing has been developed. The 0^- meson state has been studied within this model space. Considerable $q\bar{q}$ and $(q\bar{q})^2$ mixing has been found. The first excited state is in the energy range ~1.5 GeV. This state may be relevant to the new discovered exotic meson states.

Keywords Meson states, Meson model, QCD, Exotic quark

1 Introduction

QCD is believed to be the fundamental strong interaction theory even though there is not a real verification in the low energy region. The quark- gluon confinement property of QCD does not rule out the existence of exotic quark systems, such as $(q\bar{q})^2$, $q\bar{q}g$, gg, $q^3q\bar{q}$, q^3g, q^6 etc., which are different from the $q\bar{q}$ mesons, q^3 baryons and the nuclei. The discovery of these exotic quark systems is a very important support of QCD on one hand, and will show us the low energy property of QCD on the other hand.[1] In principle there should be states with real exotic quantum numbers.[2,3] If these states can be identified, it is the most unambiguous evidence of the exotic quark systems. Unfortunately, the signals of real exotic states are very sparse up to now. However due to the development of the experimental techniques, very serious candidates of the exotic quark systems with the normal quantum numbers of the meson states have been cumulated.[4-6] These states are exotic not due to the exotic quantum numbers, but due to their exotic properties. Because these states have the same quantum numbers as those of the normal meson states, they are mixed with normal $q\bar{q}$ configuration in a relativistic theory. Therefore, it is important to have a systematic reliable calculation of the meson states in an extended Fock space

$$|M\rangle = C_0|q\bar{q}\rangle + C_1|(q\bar{q})^2\rangle + C_2|q\bar{q}g\rangle + C_3|gg\rangle + \cdots \tag{1}$$

to provide a theoretical background to replace the pure $q\bar{q}$ configuration calculation[1] for the identification of the exotics.

2 Theoretical method

As the first step along this direction, a meson model with $q\bar{q}$ and $(q\bar{q})^2$ mixing has been developed. A meson state with quantum number α is expressed as

$$|M_\alpha\rangle = \sum_i C_{i\alpha}|(q\bar{q})_i : \alpha\rangle + \sum_{ij} C_{ij\alpha}|(q\bar{q})_i(q\bar{q})_j : \alpha\rangle \tag{2}$$

where the first summation consists of three components $(u\bar{u}, d\bar{d}, s\bar{s})$, which are all valence quark, and the second one includes fifteen distinct configurations with an additional $(q\bar{q})$ pair in addition to the valence quarks. They are all coupled to the quantum numbers of the pseudoscalar mesons. Due to the negative intrinsic parity of the $(q\bar{q})$ pair, we have to assume an odd angular momentum l (=1 in our case) on the relative coordinate between two $q\bar{q}$ pairs. The possible P-wave meson excitation is not taken into account since it is well known

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from hadron spectrum calculations of Godfrey and Isgur [7] that the lowest bound states are predominantly S-wave. Each of $(q\bar{q})$ pairs is color singlet and the radial wave functions are taken to be harmonic oscillator functions, whose width parameters $b_{\rm i}$, $b_{\rm r} = 0.65$fm, with $b_{\rm i}$ being width parameter in the interior of $(q\bar{q})$ and $b_{\rm r}$ the width parameter between two $(q\bar{q})$ pairs.

The Hamiltonian is chosen to be

$$H = \sum_i \frac{p_i^2}{2m_i} + \sum_{i\neq j} V_{ij}^{\rm C} + \sum_{i\neq j} V_{ij}^{\rm G} + \sum_{i\neq j} V_{ij}^{\rm A} + \sum V_{ij}(q - qq\bar{q}) + H.C. \tag{3}$$

where

$$V_{ij}^{\rm C} = -a\vec{\lambda}_i \cdot \vec{\lambda}_j r_{ij}^2 \tag{4}$$

$$V_{ij}^{\rm G} = \alpha_s \frac{\vec{\lambda}_i \cdot \vec{\lambda}_j}{4}[\frac{1}{r_{ij}} - \frac{\pi}{2}(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{4}{3m_i m_j}\vec{\sigma}_i \cdot \vec{\sigma}_j)\delta(\vec{r}_{ij}) + \cdots] \tag{5}$$

$$V_{ij}^{\rm A} = \alpha_s \frac{(\vec{\lambda}_i - \vec{\lambda}_j^*)^2}{4}(\frac{1}{3} + \frac{f_i \cdot f_j^*}{2})(\frac{\vec{\sigma}_i + \vec{\sigma}_j}{2})^2 \frac{2\pi}{3(m_i + m_j)^2}\delta(\vec{r}_{ij}) \tag{6}$$

$$V_{ij}(q - qq\bar{q}) = i\alpha_s \frac{\vec{\lambda}_i \cdot \vec{\lambda}_j}{4}\frac{1}{2r_{ij}}[((\frac{1}{m_i} + \frac{1}{m_j})\vec{\sigma}_j + \frac{i\vec{\sigma}_j \times \vec{\sigma}_i}{m_i}) \cdot \frac{\vec{r}_{ij}}{r_{ij}^2} - \frac{2\vec{\sigma}_j \cdot \vec{\nabla}_i}{m_i}] \tag{7}$$

For a system without the particle number conservation, one should use the second quantization formalism. Here we still use the first quantization formalism, then the Hamiltonian (Eq.3) should be understood as in different Hilbert space, it includes different numbers of particles. The pure $q\bar{q}$ configuration meson model is quite successful[7], therefore in our calculation we will keep as much as possible the ingredients of that model. $V_{ij}^{\rm C}$ and $V_{ij}^{\rm G}$ are the phenomenological quadratic confinement and the effective one gluon exchange potentials, respectively. The annihilation interaction $V_{ij}^{\rm A}$ has been extended to three flavors. The real new ingredient is the $(q - qq\bar{q})$ transition interaction $V_{ij}(q - qq\bar{q})$, which is the mixture of $q\bar{q}$ and $(q\bar{q})^2$ configurations.

3 Calculation and results

The model parameters are taken as follows: the mass of u -quark is considered to be the same as that of d -quark, $m_d = m_u = 350$ MeV, the s-quark mass $m_s = 600$ MeV, the coupling constant of one gluon exchange interaction $\alpha_{\rm s} = 1.39$, the phenomenological quadratic confinement potential with a strength parameter a=25.13 MeV$\cdot$fm^{-2}.[8,9]

In the model space spanned by the colorless non-strange and strange mesons, meson pairs are shown in the first column in Table 1, matrix elements of the Hamiltonian (Eq.3) have been calculated. After diagonalization of the matrix of Hamiltonian, eigenvecters and eigenvalues are obtainted. The first three eigenvalues are 135, 151 and 1076 MeV, which correspond to the masses of π^0, η and η', respectively, and the corresponding eigenvectors are listed in the following three columns in Table 1. In our model the difference between the masses of π^0 and η are not large enough relative to the experimental ones. The quark structures of them are about the same, different only in the isospin. The one gluon exchange interaction (Eq.3), which does not depend on the flavour, can not split the masses of π^0 and η evidently. Note that our bare "η" does not contain any strange component, which is explicitly taken into account as $s\bar{s}$. From Table 1 one can easily read off all the conservation laws for the different mesons. For example, considering lines 1 and 2 and adding the components multiplied by the appropriate coefficients one sees that the physical pion contains only a pion and no η, whereas the physical η and η' contain only an η. Since we use the same mass for the u- and d-quarks, isospin is still a good quantum number, which is demonstrated by the pion containing only isospin 1 component and the η's isospin 0. All of 18 components have no definite G-parity, some combinations of them will give rise to definite G-parity, which are listed in Table 2. The numbers in the parentheses in Table 2 represent the ordinal number of a line in Table 1. As an example, (1)-(2) represents the combination of lines 1 and 2 with an appropriate coefficient 1 and (-1). It is also easy to see that physical pion contains only those components with negative G-parity, whereas η and $\eta\prime$ only those with positive G-parity.

Table 1 Structure of physical mesons. The amplitudes have been multiplied by 10^4

	mixed mesons	π^0	η	$\eta\prime$
1	$\pi+\eta$	6756	-6813	95
2	$\pi-\eta$	6756	6813	-95
3	$s\bar{s}$	0	36	9442
4	$(\rho^0+\omega)(\rho^0+\omega)$	1284	-1305	32
5	$(\rho^0-\omega)(\rho^0-\omega)$	-1284	-1305	32
6	$(\rho^0-\omega)(\rho^0+\omega)$	0	-12	1
7	$\rho^+\rho^-$	0	1317	-33
8	$\rho^+\pi^-$	-1317	0	0
9	$\rho^-\pi^+$	1317	0	0
10	$K^{*+}K^{*-}$	627	-635	1076
11	$K^{*0}\bar{K}^{*0}$m	627	635	-1076
12	K^+K^{*-}	540	-552	-1102
13	$K^0\bar{K}^{*0}$	540	552	1102
14	K^-K^{*+}	-540	552	1102
15	$\bar{K}^0K^{*0}$	-540	-552	-1102
16	$(\rho^0+\omega)\phi$	-3	3	-11
17	$(\rho^0-\omega)\phi$	-3	-3	11
18	$\phi\phi$	0	4	1909

Table 2 The combinations with definite G-parity

Negative G-parity	Positive G-parity
(1)+(2)	(1)-(2)
(4)-(5)	(3), (6), (7), (18)
(8),(9)	(4)+(5)
(10)+(11)	(10)-(11)
(12)-(15)	(12)+(15)
(13)-(14)	(13)+(14)
(16)+(17)	(16)-(17)

4 Conclusion

The most interesting feature of this result is the following: (1) the $q\bar{q}$ and $(q\bar{q})^2$ are mixed considerably even for the ground states of mesons, (2) the first few excited states are in the energy range $\sim$ 1.5 GeV, this is quite different from the baryon case, where, after mixing the $q^3q\bar{q}$ with q^3 configuration, the first excited states are all higher than 3 GeV [10]. This fact may be related to the observation that while there are serious candidates of exotic meson states in the energy range of 1.5 GeV, there is not any serious candidates of the exotic baryon in energy range 1~3 GeV.

As mentioned before this is just the first step to develop a hadron model to mix the $q\bar{q}$ and gluon excitation in the minimum valence configuration $q\bar{q}$ (for meson) and q^3 (for baryon) for the study of exotic quark systems, there are lots to be done in future, to enlarge the Fock space, to improve the model Hamiltonian, especially to take into account the different confinement interaction of interhadron and intrahadron, to include the open channels coupling effects, etc.

References

1 Isgur N. In: Vautherm D *et al* eds. Hadrons and hadronic matter. New York: Plenum Press, 1990; 21

2 Laudsberg L G. Sov Phys Uspekhi, 1990; 33:169.

3 Burnett T H, Sharpe S R. Ann Rev Nucl Part Sci, 1990; 40:327

4 Barnes T. In: Baryons 95, 7th International Conference in the Structure of Baryons, Oct. 3-7 1995; Sunta-Fe

5 Chung S U. In: Baryons 95, 7th International Conference in the Structure of Baryons, Oct. 3-7 1995. Sunta-Fe

6 Klempt E. In: Baryons 95, 7th International Conference in the Structure of Baryons, Oct. 3-7 1995. Sunta-Fe

7 Godfrey S, Isgur N. Phys Rev, 1985; D32:189

8 Yu You-Wen, Zhang Zong-Ye. Nucl Phys, 1984; A426:557

9 Wang Fan, Wu Guang-Han, Teng Li-Jian *et al.* Phys Rev Lett, 1992; 69: 2901

10 Su W N, Qing D, Wang F, *et al.* Los Alamos preprint LA-UR-96-94