Three approaches to assign the spins of rotational bands*

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Abstract Using the three approaches for the spin assignment of rotational bands, the observed superdeformed (SD) bands of even-even nuclei in the $A \sim 190$ region are analyzed. For the yrast SD bands ^{194}Hg(1) and ^{194}Pb(1), the spin assignments agree with the experimental results. The spin assignments for the yrast SD bands of even-even nuclei in the $A \sim 190$ region by using the three approaches are consistent with each other and are believed to be reliable.
Keywords Superdeformed bands, Spin assignment, Kinematic and dynamic moments of inertia

1 Introduction

Since the first discovery of the superdeformed (SD) band ^{152}Dy[1], ten years have passed. As it is very difficult to experimentally identify the link connecting SD bands and normally deformed (ND) states with known spins, the spins, parities, and exact excitation energies of SD bands until recently had not been conclusively determined. Recently, the discrete γ rays connecting states of the yrast SD band ^{194}Hg(1) to the yrast ND states with known spins have been discovered.[2] Thus, the spins and excitation energies of all members of ^{194}Hg(1) are established. Immediately, the spins and excitation energies of the yrast SD band ^{194}Pb(1) are also established. In the past few years various approaches to assign the spins of SD bands were proposed[4~11], and fortunately, it was found that consistent spin assignments (except in a few cases) of all the SD bands observed in the $A \sim 190$ mass region were obtained by these approaches. The experimentally established spins of the SD bands ^{194}Hg(1) and ^{194}Pb(1) provide a reliable test of the validity of the spin assignments for the SD bands.

2 Three approaches to assign the spins of SD bands

In various approaches to assign the spins of SD bands[4~11], there are several phenomenological expressions for rotational spectra, which have been proved to be suitable for the description of ND bands. In what follows we will investigate whether these expressions are suitable for SD bands.

Based on very general symmetry arguments and under the adiabatic assumption, Bohr and Mottelson[12] gave the following general expressions for the rotational band with $K = 0$ (K being the projection of angular momentum I along the symmetry axis) of an axially symmetric nucleus

$$E = A\xi^2 + B\xi^4 + C\xi^6 + D\xi^8 + \cdots \tag{1}$$

where $\xi = \sqrt{I(I+1)}$. For $K \neq 0$ band, ξ should be replaced by $\xi = \sqrt{I(I+1) - K^2}$. Extensive analyses for ND bands in rare-earth and actinide nuclei show that, $B/A \sim 10^{-3}$, $C/A \sim 10^{-6}$, $D/A \sim 10^{-9}$, i.e., the convergence of the $I(I+1)$ expansion is satisfactory. For SD bands, the convergence is even better, $B/A \sim 10^{-4}$, $C/A \sim 10^{-8}, \cdots$, i.e., a SD band is a better rigid rotator.

Another usually adopted expression for rotational spectra is the Harris' ω^2 expansion,[12,13]

$$E = \alpha\omega^2 + \beta\omega^4 + \gamma\omega^6 + \delta\omega^8 + \cdots \tag{2}$$

where $\omega = \mathrm{d}E/\mathrm{d}\xi$ is the angular frequency. There exist evidences that the convergence of the ω^2 expansion seems to be superior to the $I(I+1)$ expansion.[12] In particular, the two-parameter Harris expansion

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$$E = \alpha\omega^2 + \beta\omega^4 \tag{3}$$

is widely used to investigate nuclear high spin states. It was demonstrated[14] that this expression is equivalent to the variant moment of inertia model.[15]

The following abc expression for rotational spectra was derived from the Bohr Hamiltonian (including the higher order quadrupole deformation term $k\beta^4$) for a nucleus with small axial-asymmetry ($\sin^2 3\gamma \ll 1$)[16,17]

$$E(I) = a\left[\sqrt{1+b\xi^2} - 1\right] + c\xi^2 \tag{4}$$

where $c\xi^2$ is a small correction and c may be positive or negative according to the sign of k. For $c = 0$, Eq.[4] is reduced to the Holmberg-Lipas empirical formula.[18] Analyses showed that the abc expression is superior to the other expressions with 3 parameters ($ABC, \alpha\beta\gamma, \cdots$), and is especially suitable for the description of SD bands.[5~8]

In this paper three approaches based on these expressions for rotational bands will be adopted to assign the spins of SD bands. It will be shown that the same results will be obtained from the three approaches and the spin assignments are just the experimental results for the SD bands ^{194}Hg(1) and ^{194}Pb(1).

2.1 Approach I. The variation of two kinds of moments of inertia with ξ (or ω)

The kinematic and dynamic moments of inertia are defined as

$$J^{(1)}/\hbar^2 = \xi\left(\frac{\mathrm{d}E}{\mathrm{d}\xi}\right)^{-1}, \quad J^{(2)}/\hbar^2 = \left(\frac{\mathrm{d}^2E}{\mathrm{d}\xi^2}\right)^{-1} \tag{5}$$

According to the expressions (1-4) for rotational bands ($K \neq 1/2$), the following five rules for the variation of $J^{(1)}$ and $J^{(2)}$ with angular momentum (or angular frequency) can be drawn (except in the bandcrossing region):

(a) As $I \to 0$, both $J^{(1)}$ and $J^{(2)}$ of a rotational band tend to the same limiting value.

(b) Both $J^{(1)}$ and $J^{(2)}$ monotonously increase with I (for $B < 0$), or monotonously decrease with I (for $B > 0$), but the slope of $J^{(2)}$ is much steeper than that of $J^{(1)}$ ($\mathrm{dln}J^{(2)}/\mathrm{d}\xi \approx 3\mathrm{dln}J^{(1)}/\mathrm{d}\xi$ in the low spin range).

(c) $J^{(1)} - \xi$ and $J^{(2)} - \xi$ plots never cross with each other.

(d) As $I \to 0$, both the slopes of $J^{(1)}$ and $J^{(2)}$ tend to zero, i.e., both $J^{(1)} - \xi$ and $J^{(2)} - \xi$ plots become horizontal as $I \to 0$.

(e) Both $J^{(1)} - \xi$ and $J^{(2)} - \xi$ plots concave upwards (for $B < 0$), or concave downwards (for $B > 0$).

$J^{(1)}$ and $J^{(2)}$ values can be extracted from the observed transition energies E_γ by using the following relations:

$$J^{(1)}(I-1)/\hbar^2 = (2I-1)/E_\gamma(I \to I-2) \tag{6}$$

$$J^{(2)}(I)/\hbar^2 = \frac{4}{\Delta E_\gamma(I)} = \frac{4}{E_\gamma(I+2 \to I) - E_\gamma(I \to I-2)} \tag{7}$$

The extracted $J^{(1)}$ depends on the spin assignment. Extensive analyses show that for all observed ND bands (except K=1/2 bands) whose spins have been established experimentally, all the five rules (a-e) hold without exception. Therefore, various $J^{(1)} - \xi$ and $J^{(2)} - \xi$ plots may be constructed for different spin assignments for a SD band, and violation of any of these five rules implies that such a spin assignment is wrong.

2.2 Approach II. Comparison of the calculated and observed transition energies

The measured γ transition energy $E_\gamma(I+2 \to I)$ may be used to fit the abc expression (4) $E_\gamma(I+2 \to I) = E(I+2) - E(I)$. The values of the parameters a, b and c can be determined by the least square fitting. It is found that for a correct spin assignment the root mean square deviation χ of the calculated E'_γs from the experimental results becomes minimum, and deviation of even ± 1 from the correct spin assignment will cause radical increase of χ by an order of magnitude. For all the ND band, the spin assignments obtained by using this approach agree with the experimental results without exception. Therefore, it is believed that this approach is also applicable to assign the spins of a SD band. It is worthwhile to mention that, while the γ transition energies E'_γs are measured rather precisely, the errors in the differences in the successive transitions, $\Delta E_\gamma = E_\gamma(I+2 \to I) - E_\gamma(I \to I-2)$, are relatively rather large. Therefore, this approach seems more reliable than that by Becker *et al.*,[4] which use the Harris' three-parameter expression for the dynamical moment of inertia to fit the experimental results.

2.3 Approach III. Systematic of the calculated bandhead moments of inertia

Because of the strong pairing correlation in ND nuclei, the bandhead moments of inertia of the ground bands of even-even nuclei varies smoothly with mass number and there exists regular systematics. For SD bandhead in a particular mass region, at least in the heavy mass A ~240 region (fission isomeric states) and A ~190 region, there is reason to believe that the bandhead moments of inertia of the yrast SD bands in neighboring even-even nuclei also vary smoothly with mass number. The extracted bandhead moment of inertia depends sensitively on the spin assignment. For example, according to Ref.[4], the corresponding kinematic and dynamic moments of inertia

$$\hbar^2/J^{(1)} = ab[1 + bI(I+1)]^{-1/2} + 2c \qquad (8)$$

$$\hbar^2/J^{(2)} = ab[1 + bI(I+1)]^{-3/2} + 2c \qquad (9)$$

and the bandhead moment of inertia is

$$J_0/\hbar^2 = 1/(ab + 2c) \qquad (10)$$

For different spin assignment the corresponding values of the parameters a, b, c, that is, the J_0 value may be quite different. From the systematics of the extracted J_0's in a particular mass region, one may judge whether the spin assignments are reasonable.

3 Comparison between the assignment and the experimental spins of the SD bands ^{194}Hg(1) and ^{194}Pb(1)

3.1 The $J^{(1)}(\xi)$ and $J^{(2)}(\xi)$ plots for the SD bands ^{194}Hg(1) and ^{194}Pb(1) (see Fig.1) are constructed from the measured E_γ's using Eqs.(6,7). While the shape of $J^{(2)}(\xi)$ plot is unconcerned with the spin assignment, the shape of the $J^{(1)}(\xi)$ plot depends sensitively on the spin assignment. The $J^{(1)}(\xi)$ and $J^{(2)}(\xi)$ plots for five different spin assignments are displayed in Fig.1. It is seen obviously that for ^{194}Hg(1), only the spin assignment I_0 =10, $E_\gamma(I_0 + 2 \rightarrow I_0)$=254.3 keV[19], is correct, because in this

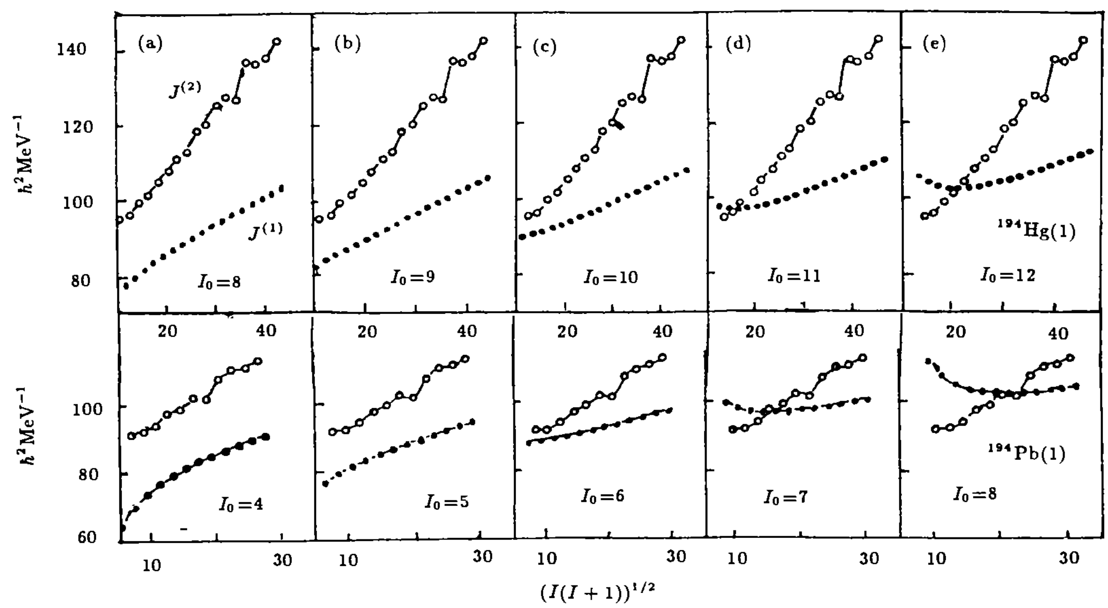

Fig.1 Variation of moments of inertia with angular moment for the SD bands ^{194}Hg(1) and ^{194}Pb(1)

case all the five rules (a-e) hold, and this spin assignment agrees with the experimental spin value.[2] Similarly, for $^{194}Pb(1)$, the spin assignment $I_0=6$, $E_\gamma(I_0+2\rightarrow I_0)=169.6$ keV[20] is correct.

3.2 The measured E_γ's of the SD bands $^{194}Hg(1)$ and $^{194}Pb(1)$ are used to fit the *abc* equation. It is found that if the experimental spin values are used, the calculated E_γ's agree excellently with the experimental results, and the relative root mean square deviation $\chi<10^{-3}$ (see Table 1), which lies within the experimental errors. In contrast, once the spin assignment deviates the experimental one even by ±1, χ increases radically by an order of magnitude (see Fig.2).

Table 1 Comparison between the calculated and the experimental $E2$ γ transition energy of yrast SD bands $^{194}Hg(1)$ and $^{194}Pb(1)$

	$^{194}Hg(1)$ $E_\gamma(I+2\rightarrow I)$, keV		$^{194}Pb(1)$ $E_\gamma(I+2\rightarrow I)$, keV	
I	expt.[19]	calc^a	expt.[20]	calc^b
6	–	–	169.6	169.5
8	–	–	213.1	213.3
10	254.3	254.3	256.4	256.4
12	296.2	296.4	298.8	298.6
14	337.7	337.6	339.7	339.8
16	377.8	377.8	380.0	380.1
18	417.1	416.9	419.1	419.4
20	455.2	455.1	458.4	457.8
22	492.3	492.2	495.6	495.4
24	528.3	528.2	531.9	532.1
26	563.6	563.3	567.9	568.0
28	597.3	597.4	603.3	603.3
30	630.5	630.6	–	–
32	662.4	662.9	–	–
34	693.8	694.3	–	–
36	725.4	725.0	–	–
38	754.6	754.9	–	–
40	783.9	784.1	–	–
42	812.9	812.7	–	–
44	841.0	480.7	–	–

a: $a=1.086\times10^4$ keV $b=0.5796\times10^{-3}$, $c=2.4983$ keV, $\chi=0.491\times10^{-3}$
b: $a=0.4252\times10^4$ keV $b=1.027\times10^{-3}$, $c=3.5261$ keV, $\chi=0.661\times10^{-3}$

It is interesting to see that the spin assignment for $^{194}Hg(1)$ and $^{194}Pb(1)$ by both approaches are consistent with each other, and agree with the experimental spin values, which implies that the usually adopted expressions (1-4) for rotational bands hold not only for ND axially symmetric nuclei, but also for SD nuclei.

Finally, let us address the bandhead moment of inertia. According to the *abc* expression (4), the bandhead moment of inertia, $J_0=\hbar^2/(ab+2c)$, depends sensitively on the spin assignment. The extracted J_0 of $^{194}Hg(1)$ is 88.6 $\hbar^2MeV^{-1}$, which is very close to 87.6 $\hbar^2MeV^{-1}$, J_0 of $^{194}Pb(1)$. Similar situation occurs in ND nuclei and fission isomeric bands. For example, for the ND ground bands $J_0(^{238}U)=68.0$ $\hbar^2MeV^{-1}$, $J_0(^{238}Pu)=66.8$ $\hbar^2MeV^{-1}$. For the fission isomeric bands, $J_0(^{236}U)\approx J_0(^{240}Pu)\approx$ 150 $\hbar^2MeV^{-1}$. On the other hand, if the assigned spin values are artificially changed by ±1, the extracted J_0 values will change significantly (see Table 2). Thus, the systematics of the extracted bandhead moments of inertia provide a valuable criteria for a spin assignment.

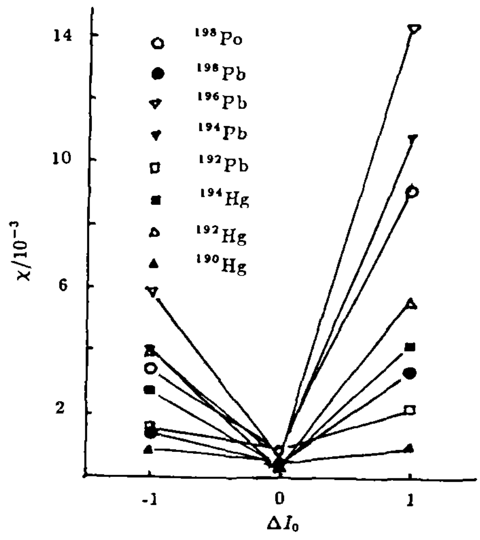

Fig.2 The root mean square deviation χ between the calculated and the experimental E_γ values of the even-even SD bands in $A\sim190$ region. The horizontal axis is the deviation between the spin assignment and the correct spin value

4 Spin assignment of the yrast SD bands of even-even nuclei in the $A\sim190$ region

The $J^{(1)}(\xi)$ and $J^{(2)}(\xi)$ plots for the other yrast SD bands $^{198}Po(1)$, $^{198}Pb(1)$, $^{196}Pb(1)$, $^{192}Pb(1)$, $^{192}Hg(1)$, $^{190}Hg(1)$ are displayed in Fig.3. The spin assignment of all the yrast SD bands observed in the $A\sim190$ region are given

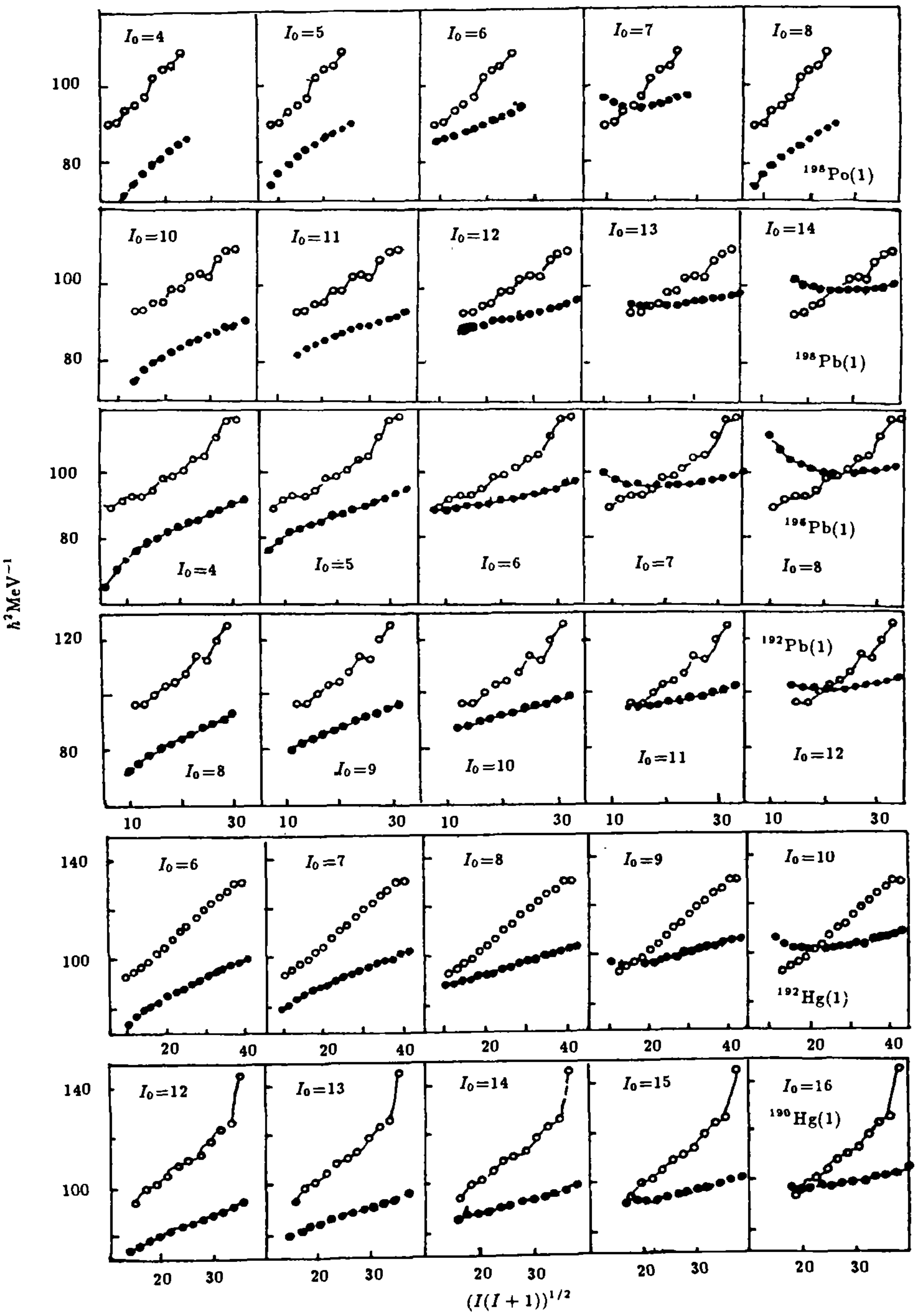

Fig.3 The variation of moments of inertia with the angular moment for the other even-even yrast SD bands in $A\sim190$ region. $J^{(1)}$ and $J^{(2)}$ are extracted from Eqs.(6,7)

in Table 2. The analysis using approach II is shown in Fig.2. It is seen that the spin assignments obtained by using approaches I and II are consistent with each other.

It is noted that the extracted bandhead moments of inertia of these yrast SD bands for correct spin assignments I_0 ($J_0 \sim$ 82-88 $\hbar^2\text{MeV}^{-1}$), are close to J_0 (^{194}Hg(1)) and J_0 (^{194}Pb(1)). However, if the spin assignments decrease by one, the extracted $J_0 \sim$ 70-80 $\hbar^2\text{MeV}^{-1}$, and if the spin assignment increase by one, the extracted J_0 $\sim$90-98$\hbar^2\text{MeV}^{-1}$. These J_0 values seem unreasonable compared with the J_0 (^{194}Hg(1)) and J_0 (^{194}Pb(1)) values extracted by using the experimental spin values.

Table 2 Spin assignments and bandhead moments of inertia of the SD bands for even-even nuclei in $A \sim 190$ region

SD band	E_γ ($I_0+2 \rightarrow I_0$) exp.(keV)	Spin assignment I_0	Bandhead moment of inertia $J_0/\hbar^2\text{MeV}^{-1}$ I_0	I_0-1	I_0+1
^{198}Po(1)	175.9[21]	6	84.2	68.9	94.5
^{198}Pb(1)	304.6[22]	12	87.2	78.3	93.9
^{196}Pb(1)	169.9[23]	6	87.8	74.3	96.3
^{194}Pb(1)	169.6[20]	6	87.6	72.8	97.4
^{192}Pb(1)	262.6[24]	10	84.8	73.8	93.7
^{194}Hg(1)	254.3[19]	10	88.6	79.9	96.1
^{192}Hg(1)	214 6[25]	8	87.2	76.8	95.5
^{190}Hg(1)	360.0[26]	14	82.3	75.2	88.9
^{194}Hg(2)	201.3[27]	8	93.6	83.1	101.1
^{194}Hg(3)	262.3[27]	11	93.9	85.0	100.7
^{194}Pb(2a)	241.2[28]	10	94.5	82.2	102.6
^{194}Pb(2b)	260.9[28]	11	94.2	83.3	107.1

In summary, the spin assignments for the yrast SD bands in the A $\sim$190 region by using the three approaches are consistent with each other, and for the SD bands ^{194}Hg(1) and ^{194}Pb(1), the spin assignments agree with the experimental determination.

References

1 Twin P J, Nyako B M, Nelson A H *et al.* Phys Rev Lett, 1986; 57:811

2 Khoo T L, Carpenter M P, Lauritsen T *et al.* Phys Rev Let, 1996; 76:1583

3 Brinkman M J, Becker J A, Lee I Y *et al.* Phys Rev, 1996; C53:R1461

4 Becker J A, Henry E A, Kuhnert A, *et al.* Phys Rev, 1992; C46:889

5 Drapper J E, Stephens F S, Deleplanque M A *et al.* Phys Rev, 1991; C42:R1791

6 Xing Z, Chen X Q. High Enger Phys & Nucl Phys (in Chinese), 1991; 15:1020

7 Zeng J Y, Meng J, Wu C S *et al.* Phy Rev, 1991; C44:R1745

8 Wu C S, Zeng J Y, Xing Z *et al.* Phys Rev, 1992; C45:261

9 Xu Fu-Rong, Hu Ji-Min. Phys Rev, 1994; C49:1449

10 Piepenbring R, Protasov K V. Z Phys, 1993; A345:7

11 Zeng J Y, Lei Y A, Wu W Q *et al.* Commun Theor Phys, 1995; 24:425

12 Bohr A, Mottelson B R. Nuclear structure, Vol II, Massachusetts: Benjamin, 1975

13 Harris S M. Phys Rev, 1965; 138B:509

14 Klein A, Dreizler R M, Das T K. Phys Lett, 1970; 31B:333

15 Scharff-Goldhaler G, Dove C, Goodman A L. Ann Rev Nucl Sci, 1976; 26:239

16 Wu C S, Zeng J Y. Commu Theor Phys, 1987; 5:51

17 Huang H X, Wu C S, Zeng J Y. Phys Rev, 1989; C39:1617

18 Holmberg P, Lipas P O. Nucl Phys, 1968; A117:552

19 Beausang C W, Henry E A, Becker J A *et al.* Z Phys, 1990; A335:325

20 Brinkman M J, Kuhnert A, Henry E A *et al.* Z Phys, 1996; A336:115

21 McNabb D P, Baldsiefen G, Bernstein L A *et al.* Phys Rev, 1996; C48:R541

22 Clark R M, Wadsworth R, Hauschild K *et al.* Phys Rev, 1994; C50:1222

23 Moore E F, Liang Y, Janssens R V F *et al.* Phys Rev , 1993; C48:2261

24 Henry E A, Kuhnert A, Becker J A *et al.* Z Phys, 1991; A338:469

25 Becker J A, Roy N, Henry E A *et al.* Phys Rev, 1990; C41:R9

26 Janssens R V F, Carpenter M P, Drigert M W *et al.* Nucl Phys, 1990; A250:75c

27 Cullen D M, Riley M A, Alderson A *et al.* Nucl Phys, 1990; A520:105c

28 Hughes J R, Becker J A, Brinkman M J *et al.* Phys Rev, 1994; C50:R1265