Dynamic stabilization of D-T burn in Tokamak reactors*

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Abstract A simple, engineeringly feasible dynamic method is supposed to control the deuterium-tritium burn process in Tokamak reactors operated in an advanced scenario. The thermal transport of the D-T plasma is described by an anomalous thermal conduction which is a radially increasing function and the central conduction value is proportional to the central temperature of the plasma. The dynamic external heating power is selected to be inversely proportional to certain power function of this temperature. As a result, the D-T burn can undergo in controllable way in different temperature regimes with different power output. Anomalous alpha particle transport effect is taken into account. It can affect the resultant plasma equilibrium, the reactor efficiency, the operation mode and so on.

Keywords Burning plasma, Thermal instability, Dynamic control, Anomalous alpha diffusion

1 Introduction

The deuterium-tritium (D-T) burn process is known to be thermally unstable due to the strong dependence of the fusion reaction rate on the ion temperature. Complete self-sustaining burn seems to be engineeringly unfeasible because the operation temperature needed is too high. It has long been recognized that some kinds of controlled method should be found for the suggested Tokamak reactors[1,2]. Many numerical codes have been developed to examine designed reactor scenarios which benefit to our understanding of burn physics. However, every code is restricted by an unfortunate fact that sofar we have had no direct self-sustaining D-T experimental data base. Extrapolation from present-day tokamak has the defect that physics processes observed are essentially different from what would happen in real reactors. One of such differences is the heating source which is a strong function of the temperature whereas in present tokamaks the external heating sources are basically uncoupled with the plasma temperature; the alpha particle behavior about which we know very poorly will affect the resultant burn process in different ways. In this respect, the anomalous alpha diffusion has attracted many theoretical attentions recently. In this paper, we proceed in as simple as possible way to discuss these points. We propose a very simple, engineeringly feasible way to control the burn process. Advanced tokamak scenario is taken as the basis of our study[3]. For the anomalous plasma thermal conduction, we also propose a simple model[4] which can reveal automatically the scaling of confinement time on the input power for case of temperature independent heating source. The anomalous alpha particle diffusion is also considered in a phenomenological way[5].

2 Physical model

We consider the alpha particle dominating heating (the nearly self-sustaining burn) with the deuterium and tritium ion temperature being almost equal to the electron's: $T = T_i = T_e$.

The main energy loss channels are the anomalous plasma thermal conduction and the electron bremsstrahlung radiation. Assuming an unchanged density profile then the energy-balance equation takes the following form

$$3n\frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}(rn\chi\frac{\partial T}{\partial r})+p_\alpha\eta_\alpha-p_{\text{rad}}+p_{\text{dyn}} \quad (1)$$

where $\chi = \chi_i + \chi_e$ is the anomalous thermal conductivity for which we assume a radially increasing function and its central value being proportional to the central temperature T_0[4]

$$\chi(r) = \chi_0 T_0(1+\chi_1 x^2), \quad x = r/a \quad (2)$$

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where a is the effective minor radius of Tokamak plasma. The alpha-particle heating power equals

$$p_\alpha = \frac{1}{4} n^2 < \sigma v >_{DT} f_{DT}^2 E_{\alpha 0} \tag{3}$$

The D-T reaction rate is a function of the ion temperature for which we use the Hively's formula[6]

$$< \sigma v >_{DT} = g(T) = \exp\left(\frac{a_1}{T^\gamma} + a_2 + a_3 T + a_4 T^2 + a_5 T^3 + a_6 T^4\right)$$

$$\gamma = 0.2935, \quad a_1 = -21.377692, \quad a_2 = -25.204054, \quad a_3 = -0.071013427$$

$$a_4 = 1.9374541 \times 10^{-4}, \quad a_5 = 4.9246592 \times 10^{-6}, \quad a_6 = -3.9836572 \times 10^{-8} \tag{4}$$

$f_{\rm DT} = (n_{\rm D} + n_{\rm T})/n_{\rm e}$ is the dilution factor of the fuel. $E_{\alpha 0}$=3.5 MeV is the alpha energy at birth. η_α is the coupling coefficient which will be determined by the alpha diffusion process and be discussed in later section. $p_{\rm rad}$ is the bremsstrahlung radiation loss power and $p_{\rm dyn}$ is the dynamic heating term which plays the role of keeping the burn temperature in a selected range. Without this dynamic heating power, it can be seen that the burn is usually unstable against thermal perturbations.[7] From consideration of engineering feasibility, we assume this term in a form

$$p_{\rm dyn} = \frac{A_{\rm d}}{T_0^{s_d}}(1 - x^2) \tag{5}$$

i.e., this power is fed back by the central temperature which is a measurable parameter. It can be seen from the later that the total power for stabilizing the burn process is a few percent of the fusion power and its changing rate is also feasible from the engineering viewpoint. To compare with other suggestions about the active control,[8] our method can keep the burn process smooth; besides, it provides the possibility to select the temperature range and then the total output of the fusion energy. These will be discussed in the following sections.

3 Dynamic control of D-T burn process

In this section, we assume the alpha particle energy is locally deposited so that everywhere we have

$$\eta_\alpha = 1 \tag{6}$$

Different from our original analysis,[7] we now use the time-dependent calculation. A subtle numerical topic occurs here that the time-dependent results are different from the equilibrium ones obtained from integrating the time-independent ordinary differential equation, that is Eq.(1) for $\frac{\partial}{\partial t}$=0. So far, we do not resolve this topic. The time-dependent calculation is usually found in literature. The complete numerical solution of this topic is waiting for experts in this respect. In the following the temperature is determined as 10 keV, n as $10^{20}\,\rm m^{-3}$ then

$$p_\alpha = 8.75 \times 10^{15} f_{DT}^2 n_{\rm e}^2 g(T)$$

$$p_{\rm rad} = 9.506 \times 10^{-2} Z_{\rm eff} n_{\rm e}^2 T^{1/2} \tag{7}$$

For most of our discussions, we do not need to clarify the concrete reactor parameters. Only an unit length of the plasma column is considered. However, for a reasonable selection of the anomalous conductivity (χ_0 in m^2/s, χ_1, a numerical factor), and for clarifying the equilibrium which is assumed to be in the advanced Tokamak regime with large fraction of bootstrap current, we need these parameters. As a baseline, we assume the following design parameters: R=6 m, a=2 m, B=6 T, $q_0 > 2$. For simplicity the density profile is given and assumed to be

$$n(x) = n_0 \exp(-\alpha x^2) \tag{8}$$

The dynamic heating power is added in the last phase of nearly self-sustaining D-T burn. The establishment of this equilibrium will be considered in another paper. We note that the eventual equilibrium is independent of the heating process as shown in Fig.1 where we obtain the same equilibrium for different initial temperatures. It is also shown that without the dynamic source, the burn would be extinguished

This depends on the response sensitivity of this source to the temperature change. From Eq.[5],

$$\frac{\mathrm{dln}p_{\mathrm{dyn}}}{\mathrm{d}t} = -s_{\mathrm{d}}\frac{\mathrm{dln}T_0}{\mathrm{d}t} \tag{9}$$

we assume that $s_{\mathrm{d}} > 2$ is feasible in engineering.

It is also noted that different operation temperatures can be approached by changing parameters A_{d}, s_{d} (Figs.2,3), this implies that the temperature variation and the output of fusion energy due to some kinds of perturbations in the density, the impurity content, the thermal conduction, etc., can be partly compensated by adjusting the dynamic source. Therefore, the total output of thermonuclear fusion energy can be maintained in some level smoothly.

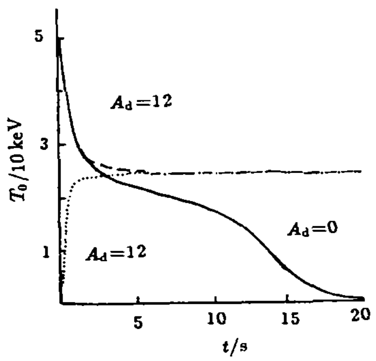

Fig.1 T_0 versus t with and without the dynamic power
Parameters: $n_0 = 2, \alpha = 2, \chi_0 = 0.4\mathrm{m}^2/\mathrm{s}, \chi_1 = 4, A_{\mathrm{d}} = 12, s_{\mathrm{d}} = 4$

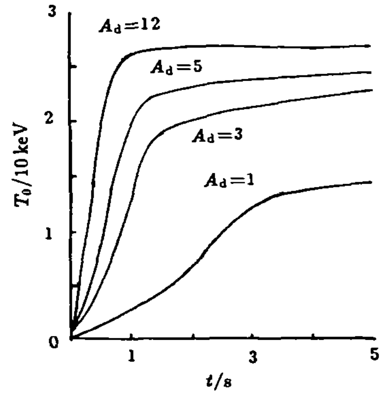

Fig.2 T_0 vs time t for $s_{\mathrm{d}} = 3$

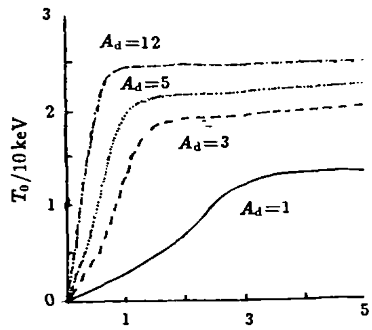

Fig.3 The variation of central temperature T_0 versus time t for $s_{\mathrm{d}} = 4$

The energy amplifier factor Q defined as the ratio of the released fusion energy to the input heating energy can be calculated from the obtained thermal equilibrium

$$Q = \frac{5\int_0^1 \mathrm{d}x x p_\alpha}{A_{\mathrm{d}}/4T_0^{s_{\mathrm{d}}}} \tag{10}$$

A typical value of Q=30 or larger is thought to be suitable for a fusion reactor.

4 Advanced Tokamak scenario

In this section, we show some merits of thermal equilibrium obtained by above method. One of these is the large fraction of the bootstrap current. When the plasma temperature and the density profiles are given, the ohmic and the bootstrap current densities are determined by[8]

$$j_{\mathrm{oh}} = j_0(T/T_0)^{3/2}(Z_{\mathrm{eff},0}\gamma_{E,0}/Z_{\mathrm{eff}}\gamma_E)$$

$$j_{\mathrm{boot}} = \frac{1.6\times10^3}{B}(\frac{R}{r})^{1/2}q[(1.67K_{13}-K_{23})\frac{\mathrm{d}T}{\mathrm{d}r} - 2K_{23}\frac{\mathrm{d}n}{\mathrm{d}r}] \tag{11}$$

where the safety factor q is determined by the Ampere's law

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(r^2/q) = \frac{4\pi}{c}(\frac{Rj}{B})$$

$$j = j_{\mathrm{oh}} + j_{\mathrm{boot}} \tag{12}$$

where Z_{eff} is the effective charge number, K_{13}, K_{23} are related to Z_{eff} through[8]

$$K_{13} = 1.46(1 + 0.67/Z_{\mathrm{eff}})$$

$$K_{23} = 3.05(1 + 0.19/Z_{\mathrm{eff}}) \tag{13}$$

As an example the temperature profile, current profile and the q profiles are shown in Figs.4~6.

$$K_{13} = 1.46(1 + 0.67/Z_{\text{eff}})$$
$$K_{23} = 3.05(1 + 0.19/Z_{\text{eff}}) \tag{13}$$

As an example the temperature profile, current profile and the q profiles are shown in Figs.4~6. Due to the large fraction of the bootstrap current, this q-profile has a natural reversed-shear region in the near magnetic axis area which is thought to be beneficial to plasma confinement.

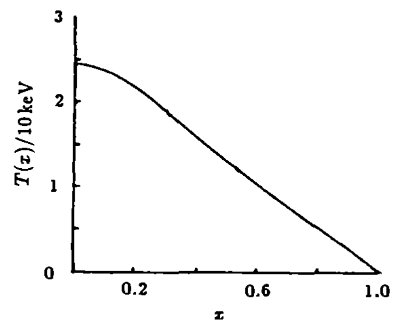

Fig.4 The temperature profile of an advanced Tokamak equilibrium a=2m, A_d=12, s_d=4

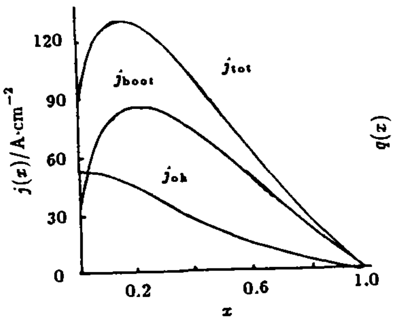

Fig.5 The current density profiles of j_{oh}, j_{boot}, j_{tot} with q_0 =3.0

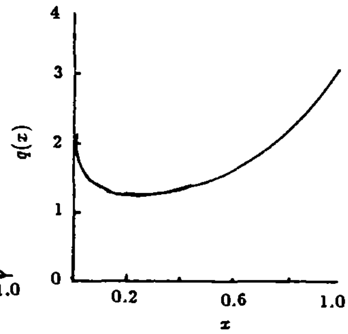

Fig.6 The q-profile with q_0=3.0

For above example, $I_{\text{boot}}/I_{\text{tot}}$= 0.736; the total fusion output power $p_{\text{fus}} = 5p_\alpha = 4\pi^2 Rka^2 \int_0^1 \mathrm{d}x x p_\alpha$ =4.58 GW, with the enhancement factor of the heating power Q=57; the central and average plasma beta value are 0.109 and 0.029, respectively. Whether this equilibrium is stable against MHD modes needs to be studied elsewhere.

5 Effect of anomalous alpha particle diffusion

So far, we only consider the case that the alpha particle energy is deposited locally. However, there are many mechanisms, of which some are experimentally observed and some are anticipated theoretically, indicating much broader deposition of alpha heating power due to anomalous alpha particle diffusions. The authors of Refs.[5,9] use a phenomenological description of this effect with prescribed diffusion coefficient. From the borne energy $E_{0\alpha}$=3.5 MeV to the ash energy, the energetic alpha particles are devided into N groups. For each group except the first one, the particle balance equation is expressed as

$$\frac{\partial n_{\alpha_j}}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}(rD_{\alpha_j} n_{\alpha_j}) + \frac{n_{\alpha_{j-1}}}{\tau_{s_{j-1}}} - \frac{n_{\alpha_j}}{\tau_{s_j}}, \quad j > 1 \tag{14}$$

τ_{s_j} is the classical slowing down time from the j-th group to the next one by collisions between the energetic alpha particles in this group and the background plasma.[5~9] For the first group, the particle balance equation is

$$\frac{\partial n_{\alpha 1}}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}(rD_{\alpha 1}\frac{\partial n_{\alpha 1}}{\partial r}) - \frac{n_{\alpha 1}}{\tau_{s1}} + n_D n_T < \sigma v >_{DT} \tag{15}$$

In this way, the coupling coefficient of the alpha particle with the background plasma η_α is determined by

$$\eta_\alpha = 1 + \frac{1}{E_{\alpha 0}}\{\sum_j(\frac{1}{r}\frac{\partial}{\partial r}(rD_{\alpha j}\frac{\partial n_{\alpha j}}{\partial r}))E_{\alpha j}\}/(n_D n_T < \sigma v >_{DT}) \tag{16}$$

The numerical method to solve above equations for the given alpha diffusion model has been discussed in Ref.[5]. They should be combined with the plasma energy balance equations(1)-(5). Results of calculation show that for the given conditions, the increase in alpha diffusion would lead to the decrease in plasma temperature and then the output of fusion energy. With the increase in alpha diffusion, the required power for dynamic heating will increase and the ratio of the total output power of fusion energy to the heating energy will decrease. Indeed, only a limited diffusion is allowable for an economic fusion reactor. This will be discussed in a separate paper.

Finally, the plasma configuration, including the bootstrap current and the central reversed shear region, are little affected by the alpha diffusion. This is simply due to the fact that the bootstrap current mainly depends on the density profile and weakly on the temperature's.

6 Conclusions

In this paper, we suggest to use a simple, engineeringly feasible dynamic heating source to control the deuterium-tritium burn in Tokamak reactors. Extending the presentday tokamak thermal conduction model to this case, we find that the burn process in an advanced Tokamak reactor can be maintained rather smoothly. The anomalous alpha particle diffusion is a critical issue which affects the resultant temperature and then the fusion-energy output greatly. In fact, an economic Tokamak reactor must rely on the effective control of alpha diffusion which should be a sticky task for future study.

References

1 Anderson D, Elevant T, Hamnen H *et al.* Fusion Tech, 1993; 23(1):5. and references therein

2 Bromberg L, Fisher J L, Cohn D R. Nucl Fusion, 1980; 20(1):203

3 Goldston R J, Batha S H, Bulmer R H *et al.* Plasma Phys Contrl Fusion, 1994; 36(1):B213

4 Shi Bing-Ren. Tokamak local transport model and scaling relation under high power heatings. SWIP report, June 1996. To be published in Chinese journal Nucl Fusion and Plasma Phys

5 Yu Ting-Yan, Shi Bing-Ren. Chinese journal Nucl Fusion and Plasma Phys, 1995; 15(3):12

6 Hively L M, Nucl Fusion, 1976; 17(6):873

7 Shi Bing-Ren, Long Yong-Xing, Zhao Hui-Guo. Chinese journal of Nucl Fusion Plasma Phys, 1996; 16(2):32

8 Hinton F L, Hazeltine R, Rosenbluth M N. Rev Mod Phys, 1976, 48(1):239

9 Sigmar D J, Gormey R, Kamelander G. Nucl Fusion, 1993; 33(5):477

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