Mathematical feature of photon spectra produced in ultra-relativistic heavy-ion collision*

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Abstract In 1994 the first single-photon spectra from the 200 GeV/A S + Au collisions at CERN SPS were reported by WA80 group. Based on these data, it can be proved that as long as there is an instantaneous thermal distribution $T(r,t)$ in an expanding fireball at each instant, the basic mathematical feature of various kinds of photon spectra is that photon yield is approximately an exponential function of the transverse momentum P_T in some region, which is basically irrelevant to the uncertainties enclosed in the theoretical estimations.
Keywords Relativistic hydrodynamics, Quark-Gluon plasma, Single -photon spectrum

1 Introduction

One of the mainest goals of the future ultra-relativistic heavy-ion collision experiments at the BNL Relativistic Heavy Ion Collider (RHIC) and the CERN Large Hadron Collider (LHC) is the production and studies of a new form of matter, the so-called quark-gluon plasma (QGP), which is expected to exist during the first few fm/c of the collisions by QCD theory.[1] And the high-energy photon was, for a long time, considered to be one of the best probes to get information about the early hot stage of heavy-ion collisions.[2] Recently the first statistically significant data on single-photon spectra for the 200A GeV S + Au collisions at the CERN Super Proton Synchrotron (SPS) were reported by WA80 group.[3] Based upon the reanalysis on the experimental data they claimed that the sources of systematic error are basically under control and at the 90% confidence leve, they have further confirmed, for central collisions the measured photon excess at each P_T, averaged over the range 0.5 GeV/$c \leq P_T \leq$ 2.5 GeV/c, corresponded to 5.0% of the total inclusive photon yield with a statistical error of σ_{stat} = 0.8% as well as a systematic error of σ_{syst} = 5.8%.[4] Although these preliminary results have generated a great deal of theoretical interest,[5−9] their tantalizing results[5−9] are not reliable conclusions. On the one hand, it seems that simple fireball models, which were based on assumption of thermal equilibrium throughout the evolution of the system as well as some adjustable higher initial temperatures $T_i \sim 250 \sim 300$ MeV, have been proved to be quite successful.[6−9] On the other hand, the experimental results have also exhibited a remarkable degree of coincidence with the predictions of the microscopic dynamical model RQMD which reveals that local equilibration is only achieved toward the end of the collision.[10] However, there is still no perfect realization to the following question: where the excess photons come from. It is well known that there are some uncertainties and insufficient knowledge in both experimental and theoretical studies.[11−12] In this case one would study how the different physical factors such as the initial temperature T_i, the critical deconfinement transition temperature T_c, the freeze-out temperature T_f, bag constant and latent heat affect the single-photon spectra rather than make up a set of parameters to fit one kind of theoretical model to the preliminary WA80 data. In this paper it will be investigated that the basic mathematical feature and its physical root of the various kinds of photon spectra produced in an ultra-relativistic heavy-ion collision.

2 High-energy photon sources

Nowdays it is generally agreed that there are three main possible sources of high-energy

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photon in the ultra-relativistic heavy-ion collisions:

(a) hard direct QCD prompt photons originating from the leading-order processes: $q(\bar{q})g \to q(\bar{q})\gamma$ Compton scattering and $q\bar{q} \to \gamma g$ annihilation, which are indeed expected to be yielded at large P_T (≥ 5GeV/c);[13-14]

(b) thermal photons around lower-P_T ($\sim 2 \sim 5$GeV/c)[13,15] originating from QGP and hadronic matter (HM), which are considered to be produced in the same processes as in the high- P_T direct photon production except with different structure functions;

(c) photons from hadronic decay of natural mesons, such as $\pi^0 \to \gamma\gamma$, $\eta \to \gamma\gamma$, $\Sigma^0 \to \Lambda^0\gamma$ and so on, as well as other hadronic processes, such as $\pi\pi \to \rho\gamma$, $\pi\rho \to \pi\gamma$ [16], $\pi\rho \to A_1 \to \pi\gamma$[17], which have been detected in the region of $P_T \sim 0.7 \sim 2$GeV/c.[17-18]

For photons originating from hadronic process $\pi\rho \to A_1 \to \pi\gamma$, Xiong *et al.*[17] suggested to use a simple parameterization analytic function to evaluate the rate of photon production

$$E\frac{dR_{\pi\rho\to A_1\to\pi\gamma}}{d^3\vec{p}} = 2.4T^{2.15}\exp(-\frac{1}{(1.35TE)^{0.77}} - \frac{E}{T}) \quad (1)$$

K. H. Kampert, the head of CERN-WA80 group, told the present author that the other two parameterized rate-equations about $\pi\rho \to \pi\gamma$ and $\pi\pi \to \rho\gamma$ processes are

$$E\frac{dR_{\pi\pi\to\rho\gamma}}{d^3\vec{p}} = 0.1434T^{1.866}\exp(-\frac{0.7345}{T} + \frac{1.45}{\sqrt{E}} - \frac{E}{T}) \quad (2)$$

$$E\frac{dR_{\pi\rho\to\pi\gamma}}{d^3\vec{p}} = T^{2.4}\exp(-\frac{1}{(2TE)^{5/2}} - \frac{E}{T}) \quad (3)$$

In Eqs.(2,3) E and T should be in units of GeV.

For the thermal photons from QGP and HM, the rates for the emission have been found surprisingly to be equal at a given lower temperature. However, because of the uncertainties and adopting different approximate models, the formulae of computing the rate of thermal photon production reported in literature are actually all different. Such as, Eqs.(4~6) are from Refs. [15, 16, 20], respectively.

$$E\frac{dR}{d^3\vec{p}} = \frac{2\alpha\alpha_s}{3.6\pi^2}T^2\exp(-E/T)(\xi + \xi^2)\ln\frac{E}{g^2T} \quad (4)$$

where ξ is the proper power of the quark suppression factor whose value $\xi \approx 1/4 \sim 1$, α the fine-structure constant, $g^2 = 5$, $\alpha_s = 0.4$, $\vec{p}$ and E denote momentum and energy of a photon produced.

$$E\frac{dR}{d^3\vec{p}} = \frac{5}{9}\frac{\alpha\alpha_s}{2\pi^2}T^2\exp(-E/T)\ln[\frac{2.912}{g^2}\frac{E}{T}] \quad (5)$$

$$E\frac{dR}{d^3\vec{p}} = \frac{\alpha\alpha_s}{3\pi^2}T^2\exp(-E/T)[\ln\frac{ET}{m_\beta^2} + c] \quad (6)$$

where $m_\beta^2 = (2\pi/3)\alpha_s T^2$, c= 1.62.

3 Photon spectrum

The photon yields are calculated by the following basic evaluating procedures.

a. Assuming a QGP fireball was formed at initial temperature T_i in a relativistic heavy-ion collision and would be subsequently cooled and eventually hadronized, the space-time evolution of a QGP system i.e. the temperature distribution function $T(r,t)$ may be evaluated by means of the conservation laws of the energy-momentum and baryon number:

$$\partial_\mu T^{\mu\nu} = 0 \quad (7)$$

$$\partial_\mu B^\mu = 0 \quad (8)$$

where $T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu + pg^{\mu\nu}$ is the relativistic stress tensor, ϵ the energy density, p the pressure, $g^{\mu\nu}$ the metric tensor(with g^{00}= -1), $u^\mu = \gamma(1, \vec{v})$ the four-velocity, $\vec{v}$ is the local flow velocity, $\gamma = (1-\vec{v}^2)^{-1/2}$, and $B^\mu = n_b u^\mu$, here n_b is the baryon number density in the rest frame.

b. Inserting $T(r,t)$ into the rate-equation of the photon production and integrating over the longitudinal momentum and over the expansion, one may obtain the transverse momentum distribution dN/dyd^2P_T of photons (or called photon yield) from QGP or HM, where y is the rapidity of a photon with momentum $\vec{p}$, P_T the transverse momentum. The above integral is similar to the following form (in central rapidity space)

$$\int_0^r r^2 \mathrm{d}r \int_0^t \exp[-a(y_{\mathrm{T}})P_{\mathrm{T}}/T]f(y_{\mathrm{T}})\mathrm{d}t \qquad (9)$$

where $y_{\mathrm{T}} = arctanhv_r$ is the transverse rapidity.

The evaluating photon spectra from HM and QGP in terms of Eqs.(1-3, 5) and Eqs.(7-9) are shown in Fig.1 and Fig.2, respectively. In the numerical evaluation, we take the initial radius of the fireball $R_0 = 3$ fm, the initial number density of the quark matter $\rho = 5\ \rho_0$, here $\rho_0 = 3 \times 0.17\ \mathrm{fm}^{-3}$. For simplicity, "the full stopping scenario" is used to evaluate the distributions of temperature in a fireball. For partially stopping case, one may introduce an effective Lorentz contraction $\gamma' = K(1-\bar{v}^2)^{-1/2}$ instead of γ, where $K = M/\sqrt{s}$, M is the invariant mass of the system, $\sqrt{s}$ refers to the energy of the fireball system in the center-of-mass frame.[21]

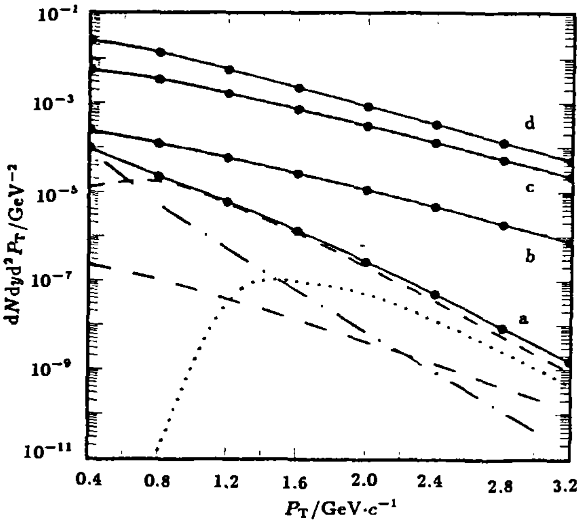

Fig.1 Transverse momentum spectra for the photons produced from several individual contributors in HM at $T_{\mathrm{i}} = 180$ MeV

Long dashed line: $\pi\pi \rightarrow \rho\gamma$, short dashed line: $\pi\rho \rightarrow \pi\gamma$, dot line: $\pi\rho \rightarrow A_1 \rightarrow \pi\gamma$, med dashed line: the main decay of natural meson $\pi^0 \rightarrow \gamma\gamma$. The solid dot curves a, b, c and d denote the total photon yields from HM including above four individual contributors for T_{i} = 180, 190, 200 and 220 MeV, respectively.

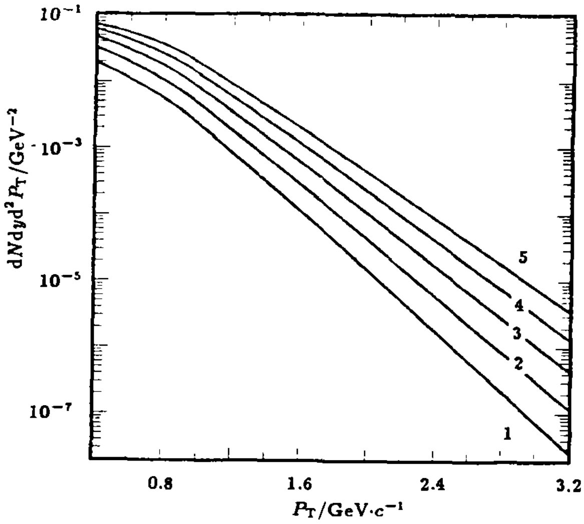

Fig.2 Thermal photon yields from QGP and HM calculated in terms of Eq.(5)

Curves 1, 2, 3, 4 and 5 correspond to T_{i} = 180, 190, 200, 210 and 220 MeV, respectively

It can be seen from Fig.1 and Fig.2 that there exists a good linearity in the region of $1 \leq P_{\mathrm{T}} \leq 3.2$ GeV/c. Therefore, it merits to study the mathematical features as well as its main physical roots of the photon spectra.

4 Mathematical feature of photon spectra

One may find that the common point in Eqs.(1~5) is all of them contain a factor $\exp(-E/T)$, which arises from statistical distribution functions (such as Boltzmann, Fermi-Dirac or Bose-Einstein statistics) and is based on assumption of local thermal equilibrium. Actually, the integral Eq.(9) may be expressed in the following general form

$$\int_a^b f(x)g(x)\mathrm{d}x \qquad (10)$$

where $g(x)$ is the exponential factor $\exp(-E/T)$

or written as $\exp[-a(y_T)P_T/T]$ in the transverse rapidity y_T. It is well known that as long as functions $f(x)$ and $g(x)$ are both integrable on an interval, for instance (a,b), and $f(x)$ is always of the same sign so that the mean value theorem for integral works, is followed.

$$\int_a^b f(x)g(x)\mathrm{d}x = g(\xi)\int_a^b f(x)\mathrm{d}x \tag{11}$$

where integral variable x lies at some point ξ of the interval (a,b) rather than at one of the end points. In most cases although Eq.(11) can not be used to evaluate Eq.(9) directly since one does not know how to choose the point ξ, however one may still use the mean value theorem for integrals to gain some important inferences, because in some cases it is enough to be able to confirm the existence of the mean value. Follow above idea, then, Eq.(9) may be reduced to

$$\exp[-a(y_T(r^*,t^*))P_T/T(r^*,t^*)] \cdot \int_0^r r^2\mathrm{d}r \int_0^t f(y_T)\mathrm{d}t \tag{12}$$

that is in form of $A\exp(-P_T/T^*)$, where $T^* = T(r^*,t^*)$ is some mean temperature for a discussed phase, A is the integrating result without the exponential factor $\exp(-E/T)$, that is

$$A = \int_0^r r^2\mathrm{d}r \int_0^t f(y_T)\mathrm{d}t \tag{13}$$

Follow the procedures in Sec.3, as one derives an instantaneous thermal distribution $T(r,t)$ for an expanding fireball, then inserting $T(r,t)$ as well as the rate-equation of the photon production into Eq.(13) we get the numerical results about A, as shown in Fig.3. Fortunately, one may find that there is a weak relation between A and P_T so that one may gain an important inference theoretically that Eq.(9) or $\mathrm{d}N/\mathrm{d}y\mathrm{d}^2P_T$, the thermal photon spectra, is approximately an exponential function of P_T in the region of $1 \le P_T \le 3.2$ GeV/c. See curves a_1 and a'_1 in Fig.3, also see Fig.2. In the same manner, one may recognize that why the total γ spectra from HM show, approximately, the exponential functions of P_T in the region of $1 \le P_T \le 3.2$ GeV/c too, but with different dropping off parameters ($-1/T^*$) in general. See curves a, b, c and d in Fig.1 again.

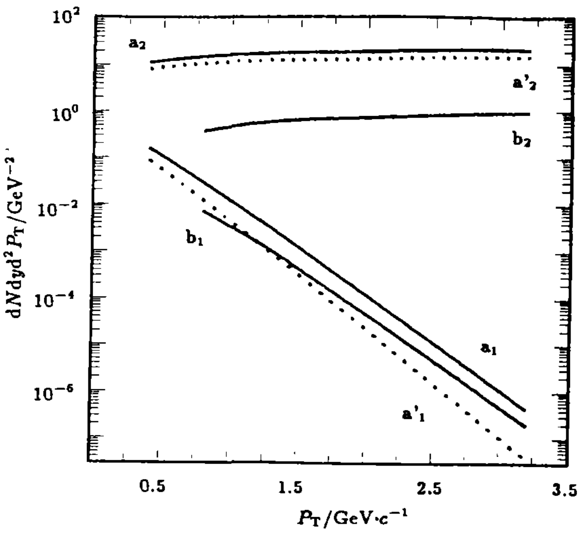

Fig.3 Evaluating results in terms of Eq.(5) and Eq.(6) are denoted by curves a and b respectively Thermal photon yields from QGP and HM at $T_i = 200$ MeV are shown by curves a_1 and b_1, curve a'_1 corresponds to $T_i = 180$ MeV. Curves a_2 and b_2 denote the calculated results without the exponential part at $T_i = 200$ MeV, curves a'_2 corresponds to $T_i = 180$ MeV

5 Discussion

As we know from above argument that the equilibration of hot and dense system created in a high-energy heavy-ion collisions plays an important role in the description of the processes. But we do not conclusively understand from the previous experimental results whether, with time, the colliding system eventually reaches some kind of local or even global thermal equilibrium so far. However the recent WA80 data show a good linearity and yield directly a parameter of temperature $T_{\exp} \approx 180$ MeV from the slope of a fitting straight line. This fact strongly implies that the measured single-photon spectra contain an exponential factor of $\exp(-P_T/T_{\exp})$. In the present paper author has already verified that as long as there is an instantaneous thermal distribution $T(r,t)$ in an expanding fireball at each instant and it is not necessary that the system becomes equilibrated in whole at some temperature, a theoretical photon spectrum may be separated into two component parts. One of them would be related to an exponential factor of exp(-P_T/T^*) and the other is approximately independent of P_T, namely the mathematical feature of photon spectra is, approximately, an exponential function of the transverse momentum P_T in some region. It is the reason why the theoretical results based on assumption of the local thermal equilibrium are in agreement with the preliminary WA80 data.[6–9] It must also be noted that

a) At present, the response time of developing photon detectors has been as fast as 1 as $\sim$ 1 fs, but the time scale during which all of the three kinds of high-energy photon are produced was expected to be a few fm/c (10^{-23} $\sim$ 10^{-22}s). Hence the measuring process, in fact, is equivalent to the process to take an average as if the integrating technique in mathematics. Hence although one is unable to conclude whether the linear behavior of WA80 data might intimate that the local thermal equilibrium has been achieved due to a rapid thermalization in the colliding system, at least, one could infer that the linear behavior of WA80 data might originate from the conditions which would be used to define the state function – temperature for an open system.

b) Although there is a large uncertainty enclosed in A, A is unable to change the shape of photon spectra but only shifts the position of the curves. In other words, the shape of the individual contribution is insensitive to the details of the very early pre-expanding stage of the system and basically irrelevant how to get the instantaneous thermal distribution $T(r,t)$ in an expanding fireball. Even if this is feature only for various individual contributions, it is a very interesting point available for diagnosing the QGP.

Above two problems are beyond the scope of the present paper and will be published separately.

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