A non-perturbation approach in temperature Green function theory*

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Abstract A set of differo-integral equations for many-body connected temperature Green's functions is established which is non-perturbative in nature and provides a reasonable truncation scheme with respect to the order of many-body correlations. The method can be applied to nuclear systems at finite temperature.

Keywords Nonperturbation, Connected temperature Green's function, many-body correlation, Truncation

The temperature Green's function theory which was originated by Matsubara in 1955,[1] has played an important role in the description of equilibrium quantum many-body systems at finite temperature.[2] However, the conventional temperature Green's function formalism is, in fact, based on a diagrammatic perturbation theory expanded in terms of interaction strength. For systems with strong interactions or repulsive hard cores (for example, nuclear system), the native perturbation theory does not work and non-perturbative methods are needed. The most obvious solution is to modify the conventional perturbation theory and combine the series expansion with a certain summation rule. In order to derive a set of non-perturbative integro-differential equations based on some infinite summation rules, one has to resort to the intuition of Feynman diagram.[3] Apparently, these non-perturbative approaches depend strongly on Feynman diagrams and their selection rules, and hence the completeness, the unification, and the systematics are lacking for such approaches.

Besides the diagrammatically perturbative method, there is another way to evaluate Green's functions, i.e., the equation-of-motion approach introduced by Martin and Schwinger. In 1959, Martin and Schwinger constructed a hierarchy of equations-of-motion for many-body Green's functions.[4] This hierarchy allows a non-perturbative treatment for the many-body problems, if a truncation scheme is provided properly. Unfortunately, the Martin-Schwinger hierarchy suffers from the weakness that it does not provide such truncation schemes by itself.[5,6]

In Refs.[7~17], a novel method called nuclear many-body correlation dynamics or quantum correlation dynamics, has been developed. The method is expanded in terms of many-body correlations and thus provides an unified and systematic approach to treat a many-body system in a non-perturbative manner. After being invented, the method has been successfully applied to explain some important nuclear experimental data from heavy ion collisions: energetic γ production and π production, dilepton cross-section;[18] two-temperature spectra of π production and preferential emission of π;[19,20] the temperature independent of decay widths of giant resonances in hot nuclei[21]; the small amplitude nuclear motion and nuclear mass dispersion.[22] The cluster expansion and truncation technique of Green functions developed in Refs.[11~17] has been applied to study the two-body correlations in pionic systems[23] and the symmetry breaking in

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Φ^4_{1+1}-theory.[24] Very recently, two-body correlation dynamics has also been used to construct the two-body correlation transport model for heavy ion collisions.[25] So far, the nuclear many-body correlation dynamics has become a powerful non-perturbation approach in nuclear physics. In this paper, along the line of nuclear correlation dynamics we will present a temperature-dependent correlation method in the framework of temperature Green's function theory.

The equilibrium many-body system at finite temperature is conveniently described by means of the grand-canonical ensemble of statistical mechanics. The statistical operator reads

$$\hat{\rho}_{\rm G} = Z_{\rm G}^{-1}\exp(-\beta\hat{K})$$

$$Z_{\rm G} = \exp(-\beta\Omega) = {\rm Tr}\,\exp(-\beta\hat{K}) \quad (1)$$

where operator $\hat{K}$ may be regarded as the grand-canonical Hamiltonian,[3] which is defined as

$$\hat{K} = \hat{H} - \mu\hat{N} = \int \hat{\psi}^\dagger(\mathrm{x})[t(\mathrm{x}) - \mu]\hat{\psi}(\mathrm{x})\mathrm{dx} + \frac{1}{2}\int\int \hat{\psi}^\dagger(\mathrm{x})\hat{\psi}^\dagger(\mathrm{x}')v(\mathrm{x},\mathrm{x}'))\hat{\psi}(\mathrm{x}')\hat{\psi}(\mathrm{x})\mathrm{dxdx}' \quad (2)$$

where $t(\mathrm{x}) = -\frac{1}{2m}\nabla^2$. $\hat{\psi}(\mathrm{x})$ and $\hat{\psi}^\dagger(\mathrm{x})$ denote fermion field operator and its conjugate in Schrödinger representation. The parameters μ and β are chemical potential and inverse temperature, respectively. Introducing a modified Heisenberg representation as follows

$$\hat{O}(\mathrm{x},\tau) = \exp(\hat{K}\tau)\hat{O}(\mathrm{x})\exp(-\hat{K}\tau) \quad (3)$$

where $\hat{O}(\mathrm{x})$ is an arbitrary operator in Schrödinger representation and τ is real. The τ-evolution of $\hat{\psi}(\mathrm{x},\tau)$ obeys

$$s(1)\hat{\psi}(1) = \int \mathrm{d}2v(1,2)\hat{\psi}^\dagger(2)\hat{\psi}(2)\hat{\psi}(1) \quad (4)$$

where the following notations have been used

$$s(1) = \frac{\partial}{\partial\tau_1} - t(x_1) + \mu,\ j = x_j = (x_j, \tau_j) \quad (5)$$

$$v(i,j) = v(x_i, x_j)\delta(\tau_i, \tau_j) \quad (6)$$

The n-body temperature Green's function is conventionally defined as

$$\mathcal{G}^{(n)}(1\cdots n; 1', \cdots n') = (-1)^n < T_\tau\hat{\psi}(1)\cdots\hat{\psi}(n)\hat{\psi}^\dagger(n')\cdots\hat{\psi}^\dagger(1') > \quad (7)$$

where T_τ is τ-ordering operator and $< \cdots >$ denotes ensemble average, i.e., $< \cdots >= {\rm Tr}(\cdots\hat{\rho}_{\rm G})$. The Green's function with equal τ-argument (for instance $\tau_i = \tau_i'$) is denfined as

$$[\mathcal{G}^{(n)}]_{\tau_i=\tau_i'} = \lim_{\tau_i'\to\tau_i^+}[\mathcal{G}^{(n)}]$$

The correlation part of n-body temperature Green's function can be separated out through the following cluster expansion

$$\mathcal{G}^{(n)}(1\cdots n; 1'\cdots n') = AS_{(n)}\sum_{k=1}^{n}\mathcal{G}_{\rm c}^{(k)}(1\cdots k; 1'\cdots k')\mathcal{G}^{(n-k)}((k+1)\cdots n; (k+1)'\cdots n') \quad (8)$$

where A denotes symmetrization of particle pairs (x_i, x'_i) and (x_j, x'_j), and S denotes antisymmetrization of variables x_i' and x'_j. In combination of the operators A and S, the repeated terms should be omitted as shown in Refs.[6~17]. It is evident that the cluster expansion (8) is highly nonlinear and $\mathcal{G}^{(n)}$ contains all possible correlations among n particles. $\mathcal{G}_c^{(n)}$ has been proved to be just the connected Green's function in the language of Feynmann diagram.[26] It is readily shown that $\mathcal{G}_c^{(n)}$ satisfies anti-periodic (for fermion system) boundary condition in the τ-region $[0, \beta]$, i.e.,

$$\left[\mathcal{G}_c^{(n)}(1\cdots n;1'\cdots n')\right]_{\tau_i=0} = -\left[\mathcal{G}_c^{(n)}(1\cdots n;1'\cdots n')\right]_{\tau_i=\beta} \tag{9}$$

which is important because it can be incorporated directly to the equations of motion for $\mathcal{G}_c^{(n)}$ by restricting their solutions to functions which automatically satisfy the boundary conditions. Furthermore, since the condition (9) is anti-periodic, the technique of Fourier series can be used.

Starting from Eq.(4), and after some complex and lengthy calculation one gets

$$s(1)\mathcal{G}_c^{(n)} = \delta_{n,1}\delta^{(4)}(1,1') + \left[TR_{(n+1)}v(1,n+1)AS_{(n+1)}\sum_{k\geq l\geq m=0}\sum\sum \mathcal{G}_c^{(k)}\mathcal{G}_c^{(l)}\mathcal{G}_c^{(m)}\delta_{k+l+m,n+1}\right]_{\mathrm{L}} \tag{10}$$

where $[\cdots]_{\mathrm{L}}$ denotes linked terms as in Refs.[7~10], and $TR_{(n+1)}$ is defined as

$$TR_{(n+1)}[\cdots] = \int \mathrm{d}(n+1)[\cdots]_{\mathbf{x}'_{(n+1)}=\mathbf{x}_{(n+1)},\tau'_{(n+1)}=\tau^{+}_{(n+1)}} \tag{11}$$

Eq.(10) constitutes a set of equations-of-motion for many-body correlation temperature Green's functions. This equation set is essentially non-perturbative. The main advantage of the above hierarchy resides in that it provides a natural and physical transparent truncation scheme with respect to the order of many-body correlations, and that each truncation leads to a non-perturbative temperature-dependent approximation within a temperature Green's function formalism.

The lowest order truncation is simply to neglect all many-body correlation, i.e., assuming $\mathcal{G}_c^{(n)}=0$ for $n \geq 2$, which leads to temperature-dependent HF approximation. A better approximation is to neglect more than three-body correlations namely

$$\mathcal{G}_c^{(n)} = 0, \quad (n \geq 3) \tag{12}$$

This truncation leads to the two-body temperature dependent correlation theory which consists of two coupled equations, one for $\mathcal{G}(1;1')$

$$s(1)\mathcal{G}(1;1') = \delta^{(4)}(1,1') + \int \mathrm{d}2v(1,2)[\mathcal{G}_c^{(2)}(1,2;1',2^+) + \mathcal{G}(1;1')\mathcal{G}(2;2^+) - \mathcal{G}(1;2^+)\mathcal{G}(2;1')] \tag{13}$$

and the other for $\mathcal{G}_c^{(2)}$

$$s(1)\mathcal{G}_c^{(2)}(1,2;1',2') = \int \mathrm{d}3v(1,3)[-\mathcal{G}^{(2)}(1,3;1',2')\mathcal{G}(2;3) \tag{14a}$$

$$+(1-P_{13})\mathcal{G}(3;3^+)\mathcal{G}_c^{(2)}(1,2;1,2') \tag{14b}$$

$$+(1-P_{13})(1-P_{1'2'})\mathcal{G}(1;1')\mathcal{G}_c^{(2)}(2,3;2',3^{\dagger})] \tag{14c}$$

Eqs.(13) and (14a~c) are an extension of temperature dependent Bethe-Salpeter equation, since they contain both ladder-diagram and ring-diagram series in a compact way. A truncation up to (14a~b) corresponds to the ladder diagram limit, which leads to the temperature dependent Bethe-Salpeter equation, while the remained terms are contributions from the ring diagrams.

The basic equation set of this theory provides a natural and physical transparent truncation scheme with respect to the orders of correlations. Furthermore each truncation leads to a non-perturbative approximation. It is turned out that the non-perturbative results of the conventional temperature Green's function theory are included in the present formalism as limiting cases.[25] Therefore, the correlation approach of temperature Green's function provides an unified and systematic method to study quantum many-body systems in equilibrium at finite temperature in a non-perturbative way.

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