Some new viewpoints in reactor noise analysis*

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Abstract It is proposed that the linearity criterion and order criterion via frequency spectrum features without any limitation of the model's phase can be used in reactor noise analysis. The time constant, natural frequency as well as the recovered transfer function of reactors can be obtained via the analyzable model based on reactor noise.

Keywords Reactor noise, Recovery of non-band-limited signal, Order determination, Linearity criterion

1 Introduction

There exists a fluctuation part in reactor output data that is called as reactor noise and contains the determinate dynamics of the reactor. Because of no input to the reactor, it is beneficial to use reactor noise in surveillance such as detecting malfunction of reactor components or monitoring changes in some specific dynamics. Since the authors prefer the later, the model that has an ability to be used in analysis is chosen at first. Then after the authors found that the output data from the Borssele reactor cannot be filtered to a purely white noise by the models with minimum phase, the authors have checked the data, calculated their third moments and drawn the conclusion that there exists non-Gaussian distribution in it.[1] Furthermore, breaking a prior limitation of minimum-phase system in reactor noise analysis, the authors gave a more precise description with minimum-phase and maximum-phase modes for the Borssele reactor.[2] On the other hand, by using this model features are separated one by one from the the complicated system and they will be helpful to further study and improvement of the reactor design.

In this paper, the authors introduce some developed items worth noticing in system identification. In section 2, the criterion for linearity of a stochastic system is given. The section 3 is about how to recover the transfer function from a series of discrete data. In section 4, the criterion to determine the model order is given via its frequency spectrum features.

2 Linearity criterion for a stochastic system

There is a relation between input x_k and output y_k via impulse response function $h(k)$

$$y_k = \sum_{i=1}^{p} h(k-i)x_i$$

Their Fourier transforms are $X(\omega)$, $Y(\omega)$ and $H(\omega)$. For a linear system, there are some functions such as power spectrum $P_y(\omega) = H(\omega)H^*(\omega)P_x(\omega)$, bispectrum $B_y(\omega_1,\omega_2) = H(\omega_1)H(\omega_2)H^*(\omega_1+\omega_2)B_x(\omega_1,\omega_2)$, where asterisks denote the complex conjugate and P_y and B_y can be estimated from data $\{y_k\}$

$$P_y(\omega_1) = F\{E[y_k \cdot y_{k+1}]\}$$

$$B_y(\omega_2,\omega_2) = F\{E[y_k \cdot y_{k+1} \cdot y_{k+2}]\}$$

where F is the Fourier transform operator and E is the mathematical expectation one. From a random input, there is

$$\frac{B_y(\omega_1,0)}{P_y(\omega_1)} = \frac{H(\omega_1)\cdot H(0)\cdot H^*(\omega_1)\beta}{H(\omega_1)\cdot H^*(\omega_1)Q} = (\beta/Q)H(0) = \text{const} \tag{1}$$

because β, Q and $H(0)$ are independent of frequency ω_1. When $\{x_k\}$ is the Gaussian white noise, $\beta=0$ and const=0.

On the other hand, even though there are a variety of non-linear systems, it can be proved that $B_y(\omega_1,0)/P_y(\omega_1)$ is not equal to zero and is dependent on ω_1 whether $\{x_k\}$ is Gaussian noise or not.

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3 Analyzable linear dynamics model[3]

In order to analyse a model corresponding to the differential equation of the system and for more convenience of engineering applications, the auto-regressive-moving-average model ARMA$(p, p-1)$ is straightly chosen.

$$y_k + a_1 y_{k-1} + \cdots + a_p y_{k-p} = x_k + b_1 x_{k-1}$$

$$+ \cdots + b_{p-1} x_{k-p+1}; \quad k = p+1, p+2, \cdots$$

or $AR(z)y_k = MA(z)x_k$ or $y_k = H(z)x_k$,

where $1/z$ is a back-shift operator and $H(z)$ is z transform function of $h(k)$. A characteristic equation is formed via $AR(z)$=0 and its solutions λ_i are the characteristic roots of the model. Due to the requirement that the dynamics of model should be the same as those of the differential equation coming from the characteristic equation $(s+\mu_1)(s+\mu_2)\cdots(s+\mu_p) = 0$. The later has its characteristic roots $\mu_i = -\ln\lambda_i/T$, where T is the sampling time interval.

There are three points worth noticing in analysis:

3.1 When λ_i is a real value, there is an inertia mode in the system and its time constant

$$\tau_i = -T/\ln\lambda_i \tag{2}$$

3.2 When λ_i, λ_{i+1} are a conjugate pair of complex numbers, there is a vibration mode and its natural frequency

$$\omega_{n,i} = \frac{1}{T}\sqrt{\frac{[\ln(\lambda_i\lambda_{i+1})]^2}{4} + [\cos^{-1}(\frac{\lambda_i + \lambda_{i+1}}{2\sqrt{\lambda_i\lambda_{i+1}}})]^2} \tag{3}$$

and damping ratio

$$\zeta_i = \frac{-\ln(\lambda_i\lambda_{i+1})}{2\omega_{n,j}T} \tag{4}$$

3.3 Recovery of the transfer function of the system[4]

According to Laplace transformation

$$H(s) = \int_0^\infty h(t)\exp(-st)\mathrm{d}t$$

being a non-band-limited function, $h(t)$ can be exactly recovered from infinite sampling values $h(kT)$ into

$$h(t) = \sum_{k=1}^{m} \frac{RT \cdot h(kT)}{RK \cdot (t-kT)}, \quad t \in [0, mT]$$

where $RT = (t-T)(t-2T)\cdots(t-mT)$, $RK = (k-1)T(k-2)T\cdots T(-T)\cdots(k-m)T$.
And the discrete function $h(kT)$ is obtained via inverse Z transformation

$$h(kT) = \frac{1}{2\pi jT}\int_0^\infty H(z)z^k\mathrm{d}z$$

Combining above two equations, one obtains

$$H(s) = \int_0^\infty \Big[\sum_{k=1}^{\infty} \frac{RT}{RK\cdot(t-kT)2j\pi T}\int_0^\infty H(z)z^k\mathrm{d}z\Big]\cdot\exp(-st)\mathrm{d}t$$

The orders of the summation and the Laplace transformation integral can be exchanged. It must be noticed that before Laplace transformation s does not exist. The s must be substituted for the corresponding one of {-μ_i}. Then

$$\begin{aligned} H(s) &= \frac{1}{2j\pi T}\int_0^\infty H(z)\mathrm{d}z\int_0^\infty \Big[\sum_{k=1}^{\infty}\frac{RT\cdot z^k}{RK\cdot(t-kT)}\Big]_{s=-\mu}\cdot\exp(-st)\mathrm{d}t \\ &= \frac{1}{2j\pi T}\int_0^\infty H(z)\mathrm{d}z\int_0^\infty \sum_{i=1}^{p}\exp(-\mu_i t)\cdot\exp(-st)\mathrm{d}t \\ &= \frac{1}{2j\pi T}\int_0^\infty H(z)\mathrm{d}z\sum_{i=1}^{p}\frac{1}{s+\mu_i} = \frac{1}{T}\sum_{i=1}^{p}\Big[\frac{H(z)(z-\lambda_i)}{s+\mu_i}\Big]_{z_i=\lambda_i} \end{aligned} \tag{5}$$

4 Order criterion via frequency spectrum features

For ARMA $(p, p-1)$ model, there are two kinds of method to determine the order p, one is based on model's residual minimization, such as F criterion, AIC criterion and FPE criterion. The other uses the order of Hankel matrix. From the simulations, both kinds often fail in higher orders because that a search sometimes goes into some local minimums of a complicated (higher order) camber when the former kind is used and that the singular value is considered as zero by the threshold and the order is underestimated for the later kind. The order criterion that the authors have developed via frequency spectrum features is based on the following reasons. If the search wants to go through the local minimums, it needs help. What can help it? $P_y(\omega)$ is one of the intrinsic attributes (second moment) of system, but it is not influenced by local minimums. The amplitude of frequency spectrum $H(\omega)$ is the square root of $P_y(\omega)$. So the frequency spectrum coming from FFT (fast Fourier transformation) can help residual minimization methods skip over local minimums. The search-order process is as follows:

4.1 Search for order

The F criterion is used to order determination in search of adequate order from 1 to $p+1$. Let

$$F = \frac{A_p - A_{p+1}}{2} / \frac{A_p}{N-2p-1} \tag{6}$$

where A_p and A_{p+1} are the residual sum of squares in order p model and order $p+1$ model, respectively. N is the number of data. When

$$F \leq F(2, N-2p-1) \tag{7}$$

the candidate of adequate order is p. Where $F(2, N-2p-1)$ denotes a F-distribution value with $N-2p-1$ degrees of freedom.

The candidate will be chosen as the true one after it passes the test of skip condition

4.2 Skip condition

The model's spectrum

$$H_m(\omega) = \frac{1 + b_1 e^{-j\omega} + b_2 e^{-j2\omega} + \cdots + b_{p-1} e^{-j(p-1)\omega}}{1 + a_1 e^{-j\omega} + a_2 e^{-j2\omega} + \cdots + a_p e^{-jp\omega}}$$

must be similar to the FFT spectrum in the positions of the most important peaks that are determined by the singular values of the Hankel matrix. When this requirement is not met, the order of the model must be forced to increase and F criterion is used again at later process until the adequate order is found. That means to skip over local minimums.

For example, the model of the Borssele reactor is identiied as ARMA (15,14)[5] and the order of the Hankel matrix is 6 with the threshold value 0.04. The frequency spectrum from FFT has three most remarkable peaks at 9.2 Hz, 12.7 Hz and 15.8 Hz indeed. Now the frequency spectrum of ARMA (15,14) has three peaks at 8.92 Hz, 12.72 Hz and 15.8 Hz that correspond to the characteristic roots $0.278 \pm j0.8844$, $-0.2469 \pm j0.8838$ and $-0.3815 \pm j0.8728$ and also has the minimum residual sum of square checked by Eq.(7).

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