Spin assignment of the first discovered superdeformed band ^{152}Dy(1)*

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Abstract Using the several criteria for the spin assignment of a rotational band based on very general argument, the spin of the lowest level observed in the yrast SD band ^{152}Dy(1) is assigned to be $I_0 = 26\,(E_\gamma(I_0+2 \rightarrow I_0) = 602.4\,\mathrm{keV})$, and it is demonstrated that spin assignments of $I_0 \leq 25$ as well as $I_0 \geq 27$ are in contradiction with these criteria.
Keywords Superdeformed band, Dysprosium-152, Spin assignment, moment of inertia

1 Introduction

Since the first discovery of the superdeformed (SD) band in ^{152}Dy, superdeformation has become a central focus of the study of nuclear structure. As yet, more than one hundred SD bands have been observed in the mass number $A \sim 190$, 150, 130 and 80 regions.[2–4] However, while the intraband transitions are easy to detect with modern Ge arrays, it is difficult to observe the link between the SD band and the normally deformed (ND) state. Therefore, the exact excitation energies, spins and parities of SD bands remain unknown. In the past few years several approaches to assign the spins of SD bands were developed.[5–12] Recently, the discrete γ rays directly connecting the states of the SD band ^{194}Hg(1) to the yrast states were discovered[13] and the excitation energies and spins of all members of this SD band were established. Fortunately, as expected, the spins thus established are in agreement with the spin assignment made in Refs.[5-12] (I_0=10, i.e., the transition E_γ=254.6 keV is assigned to be $12^+ \rightarrow 10^+$). It is found that, the SD band ^{194}Hg(1) has the same properties as the usuall ground band with $K = 0$ in ND nuclei and is the lowest band in the SD as well. Although the spin determination has been performed for only one SD band, the determination of the spin of the SD band ^{194}Hg(1) is a very important step. As pointed out in Ref.[13]: "with an approximate tenfold increase in statistics expected with the full GAMMASPHERE and EUROGAM arrays, it should be possible to find the one-step decay of most SD bands and to characterize their quantum numbers".

Now let us turn attention to the first discovered SD band ^{152}Dy(1), which has become the focus of many studies.[14–16] In Ref.[1], the spin of the lowest level observed in ^{152}Dy(1) was assigned to be I_0=22 ($E_\gamma(I_0+2 \rightarrow I_0)$ = 602.4 keV). This assignment is clearly not unambiguous. However, it was pointed out in Ref.[15] that it is unlikely that I_0 is less than 22. In Refs.[9-10], the spin assignment $I_0 = 25$ was made. Ref.[16] reported that in addition to ^{152}Dy(1), other five new SD bands ^{152}Dy(2-6) were observed and the I_0=24 was assigned to ^{152}Dy(1). In this paper, we use another more effective approach[17] to investigate the spin assignment of this first discovered SD band ^{152}Dy(1).

2 Rules of assignment

Usually, the kinematic and dynamic moments of inertia are extracted from the experimental transition energies by the difference quotients

$$J^{(1)}(I-1)/\hbar^2 = (2I-1)/E_\gamma(I \rightarrow I-2) \quad (1)$$

$$J^{(2)}(I)/\hbar^2 = \frac{4}{\Delta E_\gamma(I)}$$

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$$= \frac{4}{E_\gamma(I+2 \to I) - E_\gamma(I \to I-2)} \quad (2)$$

As a function of I the extracted $J^{(1)}$ increases with the assigned level spin, but the pattern of the extracted $J^{(2)}$ is independent of the spin assignment. This extraction is model-independent and is generally considered to be reliable provided the moments of inertia vary smoothly with the angular momentum. Recently, it was shown that, based on the very general features of rotational spectra, some simple, illustrative, but effective criteria of the spin assignment may be derived from the investigation of the I variations of $J^{(1)}$ and $J^{(2)}$. According to the very general argument of A. Bohr and B. R. Mottelson[18], the $K = 0$ rotational spectra of axially symmetric nuclei (K is the projection of angular momentum I along the symmetry axis), under the adiabatic approximation, can be expressed as the function of $I(I+1)$ and the rotational energy can be expanded as ($\xi = \sqrt{I(I+1)}$)

$$E = A\xi^2 + B\xi^4 + C\xi^6 + D\xi^8 + \cdots \quad (3)$$

The expression for the band of $K \neq 0$ [5–8,18] takes a form similar to Eq.(3), but includes a bandhead energy, and $\xi = \sqrt{I(I+1)}$ is replaced by $\xi = \sqrt{I(I+1) - K^2}$. It was known that the extensive nuclear rotational bands (below bandcrossing) can be described very well by Eq.(3). Systematic analyses of large amount of rotational spectra of ND rare-earth and actinide nuclei showed[18,19] that $|B/A| \sim 10^{-3}$, $|C/A| \sim 10^{-6}$, $|D/A| \sim 10^{-9}$, etc., i.e. the convergence of the $I(I+1)$ expansion is satisfactory. For the SD bands, the convergence is even better[5–8] ($|B/A| \sim 10^{-4}$, $|C/A| \sim 10^{-8}$), etc., which implies that the SD nucleus appears to be a more rigid rotator. The kinematic and dynamic moments of inertia derived from Eq.(3) are

$$J^{(1)}/\hbar^2 = \left(\frac{1}{\xi}\frac{\mathrm{d}E}{\mathrm{d}\xi}\right)^{-1}$$

$$= \frac{1}{2A}\left(1 + 2\frac{B}{A}\xi^2 + 3\frac{C}{A}\xi^4 + \cdots\right)^{-1} \quad (4)$$

$$J^{(2)}/\hbar^2 = \left(\frac{\mathrm{d}^2E}{\mathrm{d}\xi^2}\right)^{-1}$$

$$= \frac{1}{2A}\left(1 + 6\frac{B}{A}\xi^2 + 15\frac{C}{A}\xi^4 + \cdots\right)^{-1} \quad (5)$$

From Eqs.(3-5) and the above discussions, the following rules for the behavior of $J^{(1)}$ and $J^{(2)}$ may be drawn:

(a) As $I \to 0$, both $J^{(1)}$ and $J^{(2)}$ of a rotational band tend to the same limiting value ($\hbar^2/2A$).

(b) Both $J^{(1)}$ and $J^{(2)}$ monotonously increase with I (for $B < 0$), or monotonously decrease with I (for $B > 0$), but the slope of $J^{(2)}$ is much steeper than that of $J^{(1)}$ ($\mathrm{d}J^{(2)}/\mathrm{d}\xi \approx 3\mathrm{d}J^{(1)}/\mathrm{d}\xi$ in the low spin range).

(c) $J^{(1)} - \xi$ and $J^{(2)} - \xi$ plots never cross with each other.

(d) As $I \to 0$, both the slopes of $J^{(1)}$ and $J^{(2)}$ tend to zero, i.e., both $J^{(1)} - \xi$ and $J^{(2)} - \xi$ plots become horizontal.

(e) Both $J^{(1)} - \xi$ and $J^{(2)} - \xi$ plots concave upwards (for $B < 0$), or concave downwards (for $B > 0$).

Extensive analyses of large amount of available rotational bands (below bandcrossing) of ND nuclei whose spins were established show that these rules do hold without exception. As an illustrative example, the analysis of the ground band of ^{242}Pu is displayed in Fig.1. In Fig.1c the $J^{(1)}$ and $J^{(2)}$ are extracted by Eqs.(1) and (2) from the experimental rotational spectra using the measured spin sequence I= 0, 2, 4, $\cdots$. Fig.1c is a standard pattern of $J^{(1)} - \xi$ and $J^{(2)} - \xi$ plots of ND bands with spins well established. It is seen that all the rules (a-e) hold obviously. However, if the spin of each level is artificially increased by 1 $\hbar$ (Fig.1d) or $2\hbar$ (Fig.1e), i.e., the measured spin sequence I=0, 2, 4, $\cdots$ is replaced by I=1, 3, 5, $\cdots$ or $I = 2, 4, 6, \cdots$, while the pattern of $J^{(2)} - \xi$ remains unchanged, the pattern of $J^{(1)} - \xi$ changes significantly and all the rules fail. In particular, the extracted $J^{(1)}$ increases significantly at low spin and the $J^{(1)} - \xi$ and $J^{(2)} - \xi$ plots cross with each other. On the other hand, if the spin of each level is artificially decreased by 1 $\hbar$ (Fig.1b) or $2\hbar$ (Fig.1a), also all the rules fail. In particular, $J^{(1)} - \xi$ plot becomes concave downwards at low spin and as $I \to 0$, $J^{(1)}$ and $J^{(2)}$ do not tend to the same limit.

It should be noted that the same rules for the behavior of $J^{(1)}$ and $J^{(2)}$ can also be drawn from the Harris ω-expression of rotational spectra.[18,20]

$$E = \alpha\omega^2 + \beta\omega^4 + \gamma\omega^6 + \delta\omega^8 + \cdots \quad (6)$$

where $\omega = \mathrm{d}E/\mathrm{d}\xi$. Similarly, the same rules can also be drawn from the *abc* expression for rotational spectra[21]

$$E = a[\sqrt{1 + bI(I+1)} - 1] + cI(I+1) \quad (7)$$

which can be derived from the Bohr Hamiltonian (including an anharmonic potential term $k\beta^4$) of well-deformed nuclei with small axial asymmetry ($\sin^2 3\gamma \ll 1$). c may be positive (for $k > 0$) or negative (for $k < 0$) and usually $|c/a| \ll 1$. Analysis of experimental rotational spectra showed that the *abc* expression can fit the experimental rotational spectra of ND nuclei much better than the other three parameter expressions ($ABC, \alpha\beta\gamma, etc$) for rotational spectra.[21]

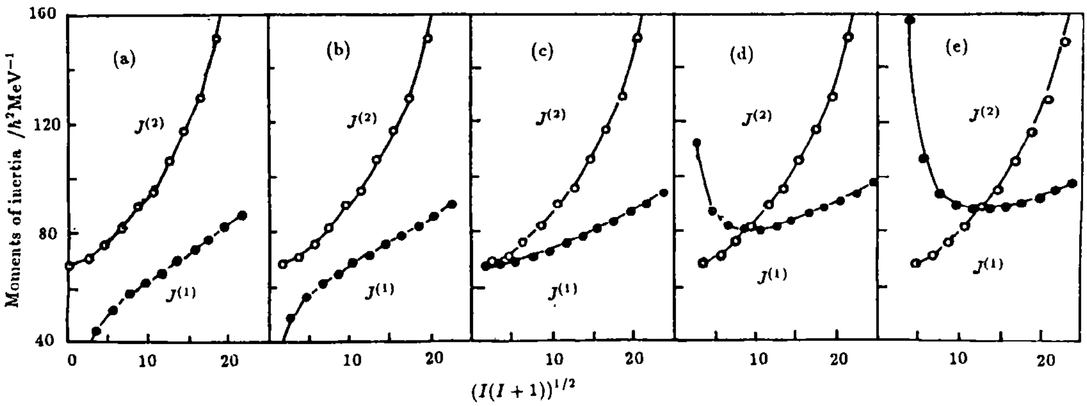

Fig.1 J$^{(1)}$ and $J^{(2)}$ of the ground rotational band of the ND nucleus ^{242}Pu

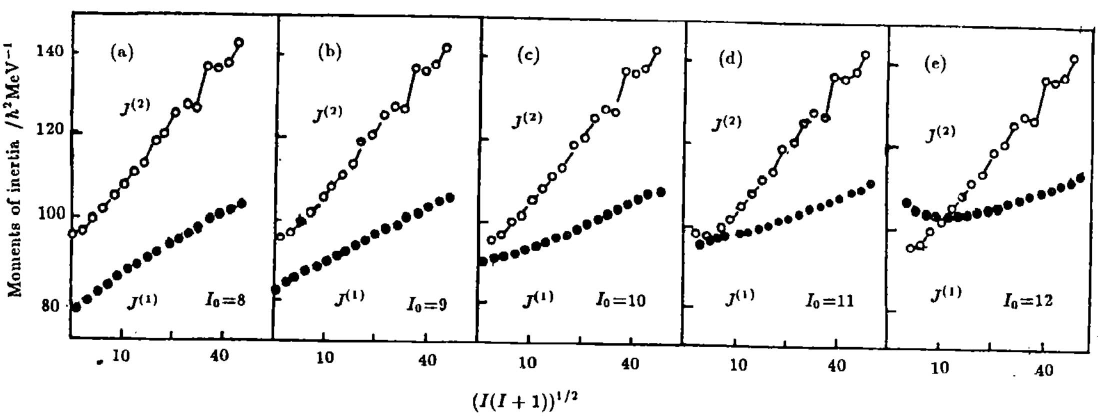

Fig.2 $J^{(1)}$ and $J^{(2)}$ of the yrast SD ^{194}Hg(1)

Now let us investigate whether the five rules mentioned above hold for the SD band ^{194}Hg(1), whose level spins have been well established recently ($E_\gamma(12 \to 10)$=254.6 keV). In Fig.2c the $J^{(1)}$ and $J^{(2)}$ are extracted by Eqs.(1,2) from the experimental transition energies and the established spin sequence. It is encouraging to see that no violation of the five

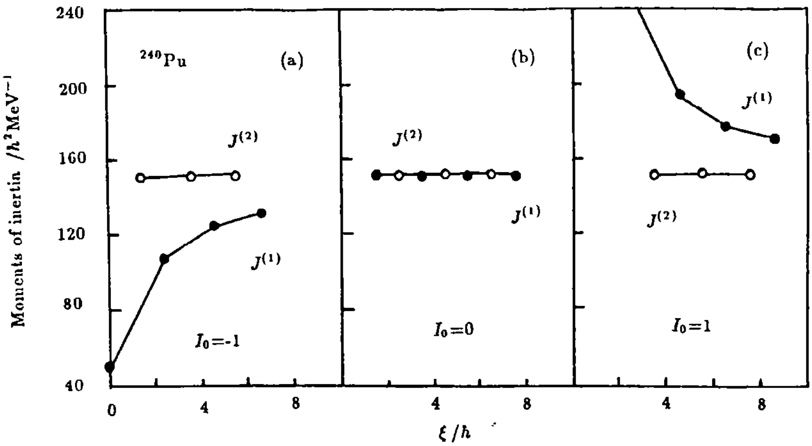

Fig.3 The superdeformed band (fission isomeric band) of ^{240}Pu
In (b), the measured spin sequence I=0, 2, 4, 6, 8 is used. In (a), the spin of each level is artificially decreased by $1\hbar$. In (c), the spin of each level is artificially increased by $1\hbar$.

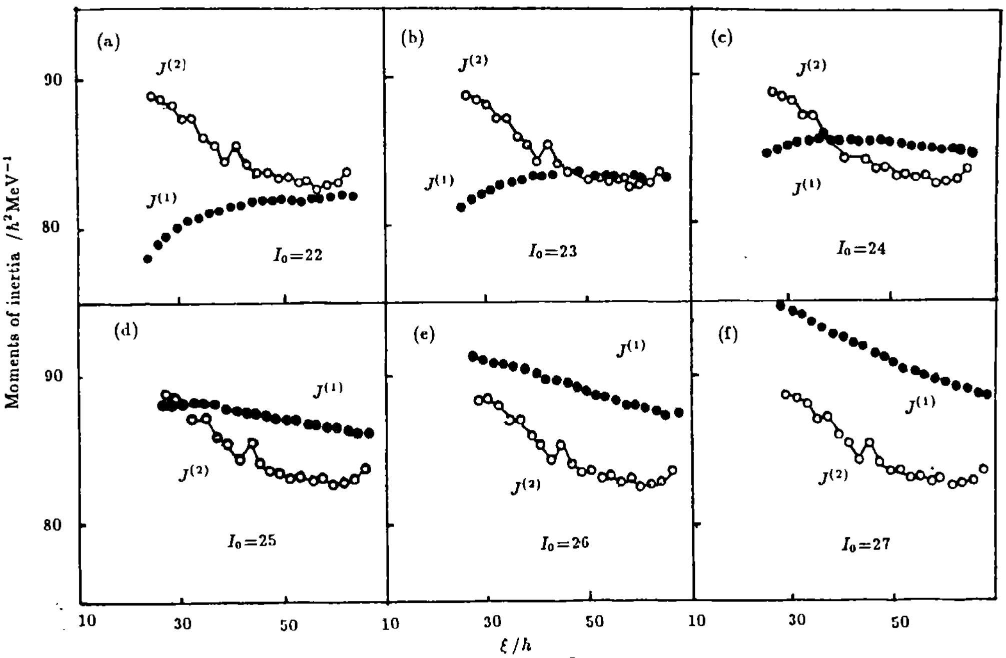

Fig.4 The kinematic (solid circle) and dynamic (open circle) moments of intertia of the yrast SD band in ^{152}Dy(1), $E_\gamma(I_0+2 \rightarrow I_0)$ =602.4 keV, extracted by Eqs.(1) and (2) from the experimental γ transition energies for various spin assignment. $\xi = \sqrt{I(I+1)}$. In Figs.4a-4f, the spin of the lowest level observed is assigned to be I_0 = 22, 23, 24, 25, 26, and 27, respectively. Obviously, only the spin assignment I_0 = 26 is allowed (Fig.4e)

rules is found. On the contrary, if the spin of each level is artificially increased by $1\hbar$ (Fig. 2d) or $2\hbar$ (Fig.2e), some of these rules fail obviously. In particular, $J^{(1)}-\xi$ and $J^{(2)}-\xi$ plots cross with each other at low spin. On the other hand, if the spin of each level is artificially decreased by $1\hbar$ (Fig.2b) or $2\hbar$ (Fig.2a), as $I\to 0$, $J^{(1)}-\xi$ does not become horizontal and the $J^{(1)}$ and $J^{(2)}$ do not tend to the same limit.

Thus, the recently established spin sequence of the yrast SD band ^{194}Hg(1) provide a solid support to the validity of the five rules (a-e) even for the SD rotational bands. In fact, another firm support has been found from the rotational bands building on the fission isomeric SD states in actinide nuclei. Fig.3 shows the similar analysis for the fission isomeric band of ^{240}Pu.[22]

3 Spin assignment

Therefore, it seems reasonable to assume that the five rules mentioned above are also applied to the SD band in the other mass regions, at least to the yrast SD band in even-even nuclei. Fig.4 displays the analysis for the first discovered SD band ^{152}Dy(1). Obviously it is seen that the spin assignments $I_0 \leq 25$ $(E_\gamma(I_0+2\to I_0)=602.4\,\text{keV})$ are in contradiction with these rules, particular rule (c), i.e., $J^{(1)}-\xi$ and $J^{(2)}-\xi$ plots cross with each other. Also the spin assignment $I_0 \geq 27$ contradicts these rules, particularly as $I\to 0$, $J^{(1)}$ and $J^{(2)}$ can not tend to the same limit. Therefore, the spin assignment $I_0=26$ is the only possible candidate for the lowest level.

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