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Approximation analytical solutions for a unified plasma sheath model by double decomposition method*

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Abstract A unified plasma sheath model and its potential equation are proposed. Any higher-order approximation analytical solutions for the unified plasma sheath potential equation are derived by double decomposition method.

Keywords Plasma sheath, Double decomposition method, Approximation analytical solution

1 Introduction

Problem of various plasma sheaths around an electrode/grid or substrate boundary has been an active topic since 1980's and received increasing attention due to its wide applications in practice of cold plasma and for commercial purpose.[1~14]

Although great progress has been achieved, profound theories, deep understanding, and unified methodologies for various plasma sheath still remain to be investigated. Several kinds of plasma sheath models have been developed, for example, for the description of the various plasma devices in low pressure gas discharge plasma[4~11], for the biased substrate in a VAD with cathode spots[1,14] and plasma-assisted vapor deposition processes, and in a negative-ion plasma and so on[12~13]. Therefore, understanding of the plasma sheath is very significant to plasma technique applications.

In this paper, we present a unified physical model which include several special kinds of plasma sheath above. Using double decomposition method, any higher-order approximation analytical solutions of the potential for the unified plasma sheath (PS) are derived.

2 A unified physical model of PS

So far several plasma sheath models for the cold plasma have been investigated.[1~15] Now, let us consider a unified physical model for various PS. Physically, characteristics of PS models in the cold plasma devices are summarized as follows: (1) When an object (such as a flat metal electrode/grid or substrate) is inserted into an unmagnetized homogeneous two- (or more) component quasineutral plasma, a PS around the object is formed. Negative or zero voltage may be applied to the object for different purposes. (2) The PS is a transition layer, in which the charge non-neutrality and the potential are suddenly decreased whereas the most region of plasma adjacent to the sheath is the quasineutral . (3) In the PS region, therefore, the ionized particles are accelerated, gain kinetic energy, then impact with the other particles and finally deposit on the object (e.g. electrode or substrate). (4) In general, the electron density obeys a Boltzmann distribution:

$$n_e = n_0 \exp(\frac{-eU}{kT_e}) \qquad (1)$$

where U is the plasma potential, n_0 the plasma density, k the Boltzmann constant and T_e the electron temperature. The electrons in the cold plasma are in local thermodynamic equilibria. (5) A shifted Maxwellian distribution of ion with a thermal energy is less than the directed energy. The ion energy includes both kinetic energy and potential energy which are conserved in the PS at any position. So $\frac{1}{2}M^+(v_i^2 - v_0^2) = -eU$, the ion velocity in the sheath is $v_i = v_0\sqrt{1-2eU/(M^+v_0^2)}$, where e is electronic charge, v_0 the thermal velocity of the ion, i.e., $v_0 = \sqrt{2\overline{Z}k_B T_e/M^+}$, where M^+ is the ion mass, $\overline{Z}$ the average charge number of the ions. (6) The distribution of the ion density can be obtained from the continuity equation of the ion

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current[4]

$$n_i = n_0 \frac{1}{\sqrt{1 - 2eU/(M^+ v_0^2)}} \tag{2}$$

For the VAD[1,14], the ion density may be of cosine form, $n_i = \frac{fI\cos(\theta)}{ev_i 2\pi\rho^2}$, where f is ion current fraction, I the arc current, v_i the ion velocity, θ for a cylindrical system is the angle between the radius vector through an arbitrary space point and the normal direction to cathode surface, and ρ the distance from the sheath edge to the cathode center. (7) Certain negative ions may be formed in the plasma[15]. Thus the negative ions become the third component in the plasma. In such a plasma, electrons would be initially expelled into the uniform ambient plasma before the any species of more heavy ions have a chance to move. This creates a localized region of net positive charge adjacent to the electrode. As time increases, the positive ions adjacent to the electrode are absorbed by the electrode or substrate. The fraction of negative ions in the plasma is γ = negative-ion density (n_0^-)/positive-ion density (n_0^+). In this case electron distribution in a quasineutral negative-ion plasma becomes $n_e^- = (1-\gamma)n_0\exp(-eU/kT_e)$. Whereas the distribution of the negative ions is of the Boltzmann equilibria

$$n^- = \gamma n_0 \exp(\frac{-eU}{kT_-}) \tag{3}$$

where T_- is the temperature of the negative ions. The positive ions are attracted to and eventually implanted in the electrode to which the negative voltage is applied. That is why these positive ions can change the surface properties of the metal electrode or substrate, and this effect has significant practical applications. This plays a very important role in the PS. (8) A few dust particles may be produced if impurity species are introduced into the plasma device and ionized. We discuss the cold positive dust ions with density n_d and having uniform charge and mass. Suppose that the dust ions are much heavier than the electrons and the working ions, on the time scale of the dust motion they can be considered to be in Boltzmann equilibria. Their densities are thus given by

$$n_d = n_{i0} \exp(\frac{-eU}{kT_i}) \tag{4}$$

where T_i is the temperature of dust ion. Of course, the dust ions (either possitive or negative) may have the similar distribution to the working ions only with different charges and masses. It is easy to deal with them in the same way. (9) For a typical discharge pressure of P = 1.33 Pa, the electron mean free path is about λ=4.82$\times$10^{-3}m[14], λ is always much larger than the Debye length (λ_d=1.3$\times$10^{-4}m), it implies that the collision can be neglected. Fig.1 shows the main picture of PS model for one dimensional case.

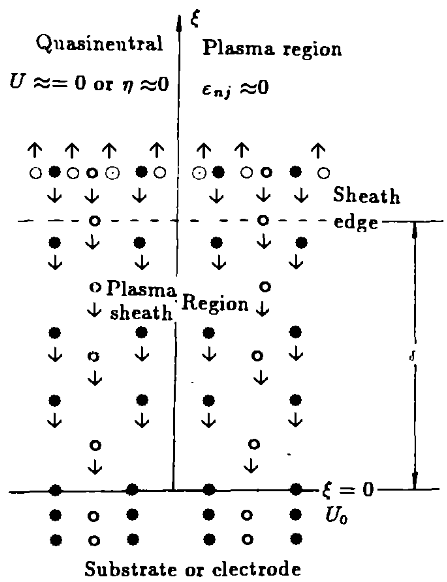

Fig.1 Main picture of plasma sheath model
○ ⊙ ● o represent electron, negative ion, positive ion and dust ion, respectively

Based on the physical picture above, the set of normalized equations we use to model a four-component plasma with a quasineutral density are described by the nonlinear equations of continuity and motion for the cold positive ions in steady state

$$\begin{cases} \nabla \cdot (n^+ \mathbf{v}^+) = 0 \\ \mathbf{v}^+ \cdot \nabla \mathbf{v}^+ = -\nabla \mathbf{U} \end{cases} \tag{5}$$

For the cold negative ions

$$\begin{cases} \nabla \cdot (n^- \mathbf{v}^-) = 0 \\ \mathbf{v}^- \cdot \nabla \mathbf{v}^- = +\nabla \mathbf{U}/M \end{cases} \tag{6}$$

For the dust ions in steady state

$$\left\{ \begin{array}{l} \nabla \cdot (n_{\rm d}\mathbf{v}_{\rm d}) = 0 \\ \mathbf{v}_{\rm d} \cdot \nabla \mathbf{v}_{\rm d} = -\frac{1}{\mu}\nabla \mathrm{U} \end{array} \right. \tag{7}$$

where $\mathbf{v}^+$, $\mathbf{v}^-$ and $\mathbf{v}_{\rm d}$ are the velocities of the working positive ion, negative ion and dust ion, respectively. n^+, n^- and $n_{\rm d}$ are the densities of the working positive, negative and dust ion, respectively. The parameter $M = M^-/M^+$ is the mass ratio of negative ion to working positive ion. The parameter $\mu = M_{\rm d}/M^+$ is the mass ratio of dust ion to working positive ion.

Thus the Poisson's equation determines the potential in terms of the densities of the four species above

$$\nabla^2 \eta = -\tilde{n}_{\rm e} + \tilde{n}^+ - \tilde{n}^- + \tilde{n}_{\rm d} \tag{8}$$

The laboratory variables are related to the normalized variables in this set of equations by

$$\left\{ \begin{array}{l} \eta = \frac{-eU}{k_{\rm B}T_{\rm e}},\ \xi = \frac{X}{\lambda_{\rm d}},\ \varsigma = \frac{Y}{\lambda_{\rm d}},\ \lambda_{\rm d} = \sqrt{\frac{\epsilon_0 k_{\rm B} T_{\rm e}}{n_0^+ e^2}} \\ \tilde{n}^- = \frac{n^-}{n_0} = \gamma \exp(-t_- \eta),\ \tilde{n}^+ = \frac{n_{\rm i}}{n_0} = \frac{1}{\sqrt{1+\beta\eta}}, \\ \tilde{n}_{\rm e} = \frac{n_{\rm e}^-}{n_0} = (1-\gamma)\exp(-\eta),\ \tilde{n}_{\rm d} = \frac{n_{\rm io}}{n_0} = \sigma \exp(t_{\rm i}\eta), \\ \alpha = H^2 \rho^{-2} \cos(\theta_{\rm b}),\ \beta = (\frac{v_0}{v_{\rm ib}})\cos(\theta_{\rm b}), \end{array} \right. \tag{9}$$

For a cylindrical system, $\xi = \frac{Z}{\lambda_{\rm d}}$ and α is a parameter introduced for the cylindrical system[14] and corresponds to the case in which the ion density has the above cosine form for the cylindrical system[1,14], but $\alpha = 1$ for the planary system. v_0 and $v_{\rm d0}$ are the thermal velocities of the working positive ion and dusty ion, respectively. $v_{\rm ib}$ is the velocity of the working positive ion at the boundary. H is the distance from the cathode to the surface of the substrate. Using the simultaneous Eqs.(5)~(9) together with Eqs.(1)-(4), and after some calculations, we can derive the normalized potential equation for a unified PS as the following

$$\frac{\partial^2 \eta}{\partial \xi^2} + \frac{\partial^2 \eta}{\partial \varsigma^2} = \alpha[-(1-\gamma)\exp(-\eta) + \sigma \exp(t_i \eta) - \gamma \exp(-t_- \eta) + \frac{1}{\sqrt{1+\beta\eta}}] \tag{10}$$

for two-dimensional planary system, and

$$\frac{1}{r}[\frac{\partial}{\partial r}(r\frac{\partial \eta}{\partial r}) + \frac{\partial}{\partial \xi}(r\frac{\partial \eta}{\partial \xi})] = \alpha[-(1-\gamma)\exp(-\eta) + \sigma \exp(t_i \eta) - \gamma \exp(-t_- \eta) + \frac{1}{\sqrt{1+\beta\eta}}] \tag{11}$$

for two-dimensional cylindrical system. To further simplify, we can reduce Eqs.(10) and (11) to one dimensional case for both planary and cylindrical systems as following

$$\frac{\partial^2 \eta}{\partial \xi^2} = \alpha[-(1-\gamma)\exp(-\eta) + \sigma \exp(t_i \eta) - \gamma \exp(-t_- \eta) + \frac{1}{\sqrt{1+\beta\eta}}] \tag{12}$$

Obviously, several special cases are included in Eqs.(10~12): (1) $\gamma \neq 0$ and $\beta \neq 0$ but $\sigma = 0$ for a negative-ion plasma. (2) $\beta \neq 0$ but $\gamma = \sigma = 0$ for normal cold plasma and so on. (3) $\sigma \neq 0$ but $\gamma = \beta = 0$ for dusty plasma, and so on. The boundary conditions on the surface of the substrate or electrode and on the plasma sheath edge for Eqs.(10)~(12) respectively are

$$\left\{ \begin{array}{l} \xi = \varsigma = 0,\ \eta(0) = \eta_0 \\ \xi = \varsigma = \delta, \eta(\delta) = \eta_{\rm b},\ \frac{\partial \eta}{\partial \xi} = 0,\ \frac{\partial \eta}{\partial \varsigma} = 0,\ \frac{\partial \eta}{\partial r} = 0 \end{array} \right. \tag{13}$$

where δ is the sheath thickness defined by the boundary potential η_b. Since the ions in the cold plasma carry kinetic energy of tens or hundreds of electronvolts, corresponding to a streaming velocity of about 10^4 m/s, which is much higher than the Bohm velocity (about 5×10^3m/s). Therefore, the potential at the sheath edge may be set to nearly zero. Thus, the physical problem of the unified PS, Eqs.(5~8), becomes to solve simultaneously Eqs.(10)/(11) or Eq.(12) together with the boundary -value conditions Eq.(13).

3 Approximate solutions of PS

In recent years, Adomain and Rach have proposed an effective analytical method – the decomposition method[14] as well as double decomposition method (DDM) for solving differential and partial differential equations which may be strongly nonlinear and stochastic cases for boundary-value problems.[17~19]

At first, we derive an approximate solution for Eq.(12) with the boundary-value conditions Eq.(13) by the DDM. In the operator format of the decomposition method[15~19], the Eq.(12) is written as

$$L\eta + R\eta + N\eta = 0 \tag{14}$$

with

$$L = \mathrm{d}^2/\mathrm{d}\xi^2, R = 0, N\eta = f(\eta) = \alpha[-(1-\gamma)f_1(\eta) + \sigma f_2(\eta) - \gamma f_3(\eta) + f_4(\eta)] \tag{15}$$

where nonlinear terms

$$f_1(\eta) = \exp(-\eta), f_2(\eta) = \exp(t_i\eta), f_3(\eta) = \exp(-t_-\eta), f_4(\eta) = \frac{1}{\sqrt{1+\beta\eta}} \tag{16}$$

where $t_i = \frac{T_e}{T_i}$ and $t_- = \frac{T_e}{T_-}$ are the normalized temperature of the dust ion and the negative ion, respectively. The nonlinearity is expressed by the Adomian polynormials as following

$$\begin{cases} f(\eta) = \sum_{n=0}^{\infty} A_n\xi^n \\ = \sum_{n=0}^{\infty}[\alpha(-(1-\gamma)a_n^1 + \sigma a_n^2 - \gamma a_n^3 + a_n^4)]\xi^n \end{cases} \tag{17}$$

where the A_n corresponds to the four Adomian's polynomials dependent on the a_n^1, a_n^2, a_n^3 and $a_n^4, n = 1, 2, ..., \infty$, respectively. So the key step is to calculate the Adomian polynomial coefficients a_n^1, a_n^2, a_n^3 and a_n^4, which can be calculated by[15]

$$a_n^i = \frac{1}{n!}\frac{\mathrm{d}^n f_i}{\mathrm{d}\eta^n} = h_n(\eta_0), i = 1, 2, 3, 4.$$

According to the algorithm above, $f_1(\eta)$, $f_2(\eta)$, $f_3(\eta)$, and $f_4(\eta)$ are easily generated. Therefore, it can be got various order components of the solution as follows

$$\begin{cases} \eta_0 = c_{0,0} + c_{1,0}\xi \\ \eta_1 = c_{0,1} + c_{1,1}\xi - \alpha[-(1-\gamma)a_0^1 - \gamma a_0^2 + \sigma a_0^3 + a_0^4] \\ \eta_2 = c_{0,2} + c_{1,2}\xi - \alpha[-(1-\gamma)a_1^1 - \gamma a_1^2 + \sigma a_1^3 + a_1^4] \\ \eta_3 = c_{0,3} + c_{1,3}\xi - \alpha[-(1-\gamma)a_2^1 - \gamma a_2^2 + \sigma a_2^3 + a_2^4] \\ \vdots \\ \eta_m = c_{0,m} + c_{1,m}\xi - \alpha[-(1-\gamma)a_{m-1}^1 - \gamma a_{m-1}^2 + \sigma a_{m-1}^3 + a_{m-1}^4]\frac{\xi^m}{m(m-1)} \end{cases} \tag{18}$$

Using the double decomposition method[17~19], the solution has the form of

$$\eta = c_0 + c_1\xi - \alpha\sum_{n\geq 2}^{\infty}[-(1-\gamma)a_{n-2}^1 + \sigma a_{n-2}^2 - \gamma a_{n-2}^3 + a_{n-2}^4]\xi^n/n(n-1) \tag{19}$$

with

$$\left\{ \begin{array}{l} c_0 = \sum_{m=0}^{\infty} c_{0,m} \\ c_1 = \sum_{m=0}^{\infty} c_{1,m} \end{array} \right. \tag{20}$$

where $c_{0,m}$ and $c_{1,m}$ are to be determined by the boundary conditions (Eq.13). It is easy to obtain the above coefficents:

$$\left\{ \begin{array}{l} c_{0,0} = \eta_0 \\ c_{0,i} = 0, i = 1, \cdots, m \\ c_{1,0} = \frac{1}{\delta}(\eta_\mathrm{b} - \eta_0) \\ c_{1,1} = \frac{1}{\delta}(\eta_\mathrm{b} - \eta_0) - \frac{\alpha\delta}{2}(a_0^1 + \sigma a_0^2 - (1-\gamma)a_0^3) \\ c_{1,2} = \frac{1}{\delta}(\phi_{\mathrm{b}]-\eta_0) - \frac{\alpha\delta^2}{3\times 2}(\mathrm{a}_1^1 + \sigma \mathrm{a}_1^2 - (1-\gamma)\mathrm{a}_1^2)} \\ \vdots \\ c_{1,m} = \frac{1}{\delta}(\phi_\mathrm{b} - \phi_0) - \frac{\alpha\delta^{m-1}}{m(m-1)}(a_{m-1}^1 + \sigma a_{m-1}^2 - (1-\gamma)a_{m-1}^2) \end{array} \right. \tag{21}$$

Thus various order approximation solutions can be expressed as

$$\left\{ \begin{array}{l} \phi_0 = c_{0,0} + c_{1,0}\xi \\ \phi_1 = \phi_0 + \eta_1 \\ \phi_2 = \phi_1 + \eta_2 \\ \phi_3 = \phi_2 + \eta_3 \\ \cdots\cdots \\ \phi_m = \phi_{m-1} + \eta_m \end{array} \right. \tag{22}$$

4 Conclusion

The basic steps of solving above PS by the DDM are as follows.

(1) Compute $c_{0,m}$ and $c_{1,m}$ from Eq.(12) by matching the solution approximates to the boundary conditions Eq.(13). (2) The Adomian polynormials, a_n^i, i=1,2,3, are generated for each nonlinearty. (3) The solution components η_n are obtained from Eq.(18), which depends on a_1^i (i=1,2,3). Since a_0^i depends only on η_0 and η_1, a_2^i depends only on η_0, η_1, η_2, etc, so η_n components are calculable step by step. (4) Compute the m-order approximation analytical solution ϕ_m from Eq.(22) and get the accuracy of the solutions when $m \to \infty$.

It is seen from the above that all of the procedures are calculable step by step and depends on the previous low-order component values. Therefore, the m-order approximation series of solutions ϕ_m can be obtained. In general, however,the 3 $\sim$ 6 order approximation solutions are satisfactory for the real PS.

In conclusion, the unified PS model is proposed and any higher-order approximation analytical solutions for the unified PS equation are obtained by the double decomposition method. The approximation analytical solutions can be used to study properties of various plasma sheaths, which includes several special cases, such as:

(1) For $\sigma = 0$ but $\gamma \neq 0$ and $\beta \neq 0$, the effects of various fractions of negative ions on the plasma sheath can be investigated[12,13] (2) For $\sigma = \gamma = 0$ but $\beta \neq 0$, the results are reduced to Refs.[4,14]. (3) For $\beta = 0$ and $\gamma = 0$ but $\sigma \neq 0$, the effects of the dust ion on the plasma sheath can be studied.

The DDM is an effective procedure to analytically solve a wide physical problems since it solves nonlinear problems rather than linearizing them, the resulting solutions are physically more realistic. It can be extended to two or more dimensional plasma boundary problems. The topic will be investigated elsewhere.

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