Application of fractal characteristic quantities of pressure fluctuation in subcooled boiling regime recognition

*KUANG Bo, XU Ji-Jun

(*Department of Nuclear Power Engineering and Automation, Shanghai Jiaotong University, Shanghai 200030*)

Abstract The dynamical behavior of the subcooled boiling two-phase system was introduced and discussed. With the introduction of fractal concept, an analysis of the fractal feature of pressure wave signals from nonlinear dynamics point of view, was carried out. Meanwhile, the pseudo phase diagrams of typical time series of sound pressure were given. Finally, through dynamic clustering and on the basis of calculating correlation dimension and Hurst exponent of pressure wave time series on different subcooling conditions, the recognition of developing regime of the two-phase system was delivered, which might provide a promising approach of recognition and diagnosis of a boiling system.

Keywords Subcooled boiling, Thermo-acoustic effect, Nonlinear dynamical characteristics, Fractal, Regime recognition

CLC numbers TK12, O359, O235 A

1 INTRODUCTION

Subcooled boiling is widely observed in both natural and forced two-phase systems, such as nuclear reactors, heat exchangers, etc. With the phase transition of the working fluid (e. g., water, freon etc.), the pressure fluctuates, which is known as the thermo-induced sound radiation phenomenon. As the boiling regime develops, both the amplitude and frequency of the pressure oscillation vary. This kind of thermo-acoustic effect can be utilized in regime recognition with the advantages of easy measurement and very little influence on the system.

In recent years, the fractal methods, which can reflect the dynamical complexity of a two/multi-phase system meticulously and comprehensively, have drawn much attention in the fields of two/multi-phase flow. For example, with the self-similarity concept of Mandelbrot, Saether *et al*[1] and Franca *et al*[2] applied the fractal technique to the detection and recognition of two-phase flow regime in tubes, in which a pressure drop was used as a detected signal. Through the fractal method suggested by Higuchi,[3] Djainal *et al*[4] made use of the fluctuation signals of temperature in the regime identification of the boiling two-phase system. In addition, it is demonstrated[5,6] that, the thermo-acoustic effect in the subcooled boiling system reflects the nonlinearity of the complex dynamical evolution process. Obviously, there exists nonlinearity, randomness and dissipativity in the subcooled boiling system. The nonlinearity comes from (1) internal "particles" of the

Manuscript received date: 2000 02 17 *E-mail:bkuang@mail1.sjtu.edu.cn

system, namely, movement of bubbles and complicated interactions among them, and (2) nonlinear external action of the system, e. g. distribution of energy and that of sound field in the system. The randomness comes from the random variation and distribution of local system conditions. As there exists strong thermodynamic nonequilibrium, viscous dissipation of the flow and sound dissipation, the dissipativity of the system appears naturally. All the above factors form the complexity and induce the fractal feature of the system dynamics.

Since the fractal feature reflects the complexity of a dynamic process within the subcooled boiling system, it is possible to utilize fractal characteristic quantities of the sound pressure signal which may indicate working condition, such as the fractional dimension values, Hurst exponents, etc. to recognize the regime dynamically in the equipment such as a water-cooled reactor. Furthermore, the method of the recognition might also be used in saturated boiling regimes.

2 DYNAMIC CHARACTERISTICS OF SOUND PRESSURE IN THE SUBCOOLED BOILING SYSTEM AND DATA ACQUISTION

2.1 Experimental setup

The test section of experimental setup is sketched in Fig.1. To control the total heat input to ensure the subcooling of bulk flow and to enhance the heat flux density on the surface of heater so that the flow superheat nearby was maintained, the heater material was specially chosen and the stainless steel tube with an i.d. of 0.25 mm, an o.d. of 0.5 mm and a length of 250 mm, was used, with the DC current of 16~20A passing through. The tubes of the loop, with an i.d. of 15 mm, were made of copper except that a plexiglass tube is applied for flow visualization just around the heater where the subcooled boiling occurs. Also, the water-cooling condensers with adequate capacity were equipped. For the objective of data acquisition, piezo-resistive pressure -transducers (p_1, p_2, p_3) and thermocouple temperature transducers (T_1, T_2, T_3) were equipped in the loop, whose locations are shown in Fig.1. Distilled and degassed water was used as working fluid and the test was performed under the current of 20A.

2.2 Data acquisition of the subcooled boiling dynamical ressure signal and phenomenological Analysis

In the experiment, oscillating pressure signals picked up by pressure transducers were transferred to A/D ports of the Computer data acquisition system (CDAS) (with the sampling rate of 27435 points per second) and through A/D transformation, the electric signals were converted to pressure data and saved in data files (which contains 4096 data in each file). The acquired data were filtered by a wavelet software filter and the time series of sound pressure for analysis were thus obtained.

2.2.1 Phenomenological analysis of the system regime based on typical pressure wave signals

Three typical groups of pressure wave time series were acquired and processed (see

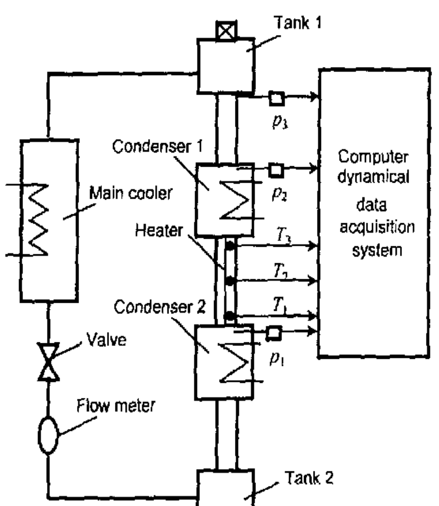

Fig.1 Test section

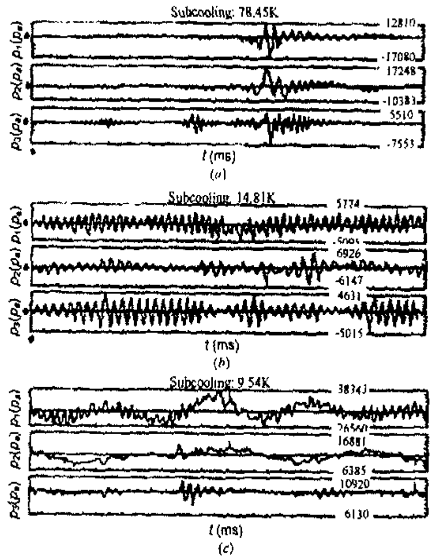

Fig.2 Typical sound pressure wave
(Average pressure: 0.1012 MPa)

Fig.2). According to the waveforms of sound pressure, the dynamic process of subcooled boiling can be roughly classified into three stages with the subcooling decreased.

(a) Initial subcooled boiling stage (subcooling range: >60 K, according to T_2)

Since very few bubbles are generated in this stage, the pressure waveform approximately reflects the sound pressure radiation feature of single bubble generation process,[6] bubble shape oscillation from the beginning of bubble ejection simultaneously forms the matching volume oscillation. The oscillations are continued till the final stage of bubble condensation and then rebound to a shock wave which leads to a decaying volume oscillation.

(b) Middle subcooled boiling stage (subcooling range: 60~10 K, according to T_2)

With larger amount of bubbles observed and smaller subcooling established, the sound waveform becomes "successive" and the high peaks of shock waves almost disappear. The reason is that the potential difference formed by the thermodynamic nonequilibrium effect during the condensation process of bubbles is now not strong enough to lead to a rebound that forms shock. Moreover, under the restraint of the sound field of bubbles cloud itself, the volume oscillation dominates the bubbles and with the interaction of Bjerknes forces[6] between them, the vapor bubbles attract each other and their agglomeration leads to combination of the bubbles. Under this condition, sound radiation frequencies of vapor bubbles tend to be identical.

(c) Near saturated boiling stage (subcooling range: under 10 K, according to T_2)

During the stage, the subcooling is small while the vapor bubbles which grow rather big oscillating on the surface of heater. It is observed that the lumps of vapor enter and strongly stir the bulk. The two-phase region becomes inhomogeneous and the sound wave with a high frequency tends to be weakened while low frequency component enlarged.

As to the average amplitude, it is natural that the transducer $\langle \tilde{p}_2 \rangle > \langle \tilde{p}_1 \rangle > \langle \tilde{p}_3 \rangle$ since the transducer p_2 is very near the two-phase region where the bubbles are generated and p_1 locates at the inlet of single-phase region. Here, $\tilde{p}_i$ ($i=1$, 2, 3) is the oscillation amplitude of the pressure at location i and "$\langle\rangle$" refers to time averaging.

2.2.2 Evidence of fractal feature in sound pressure signal

(a) Pseudophase diagram

The pseudo phase diagram drawn from the sound pressure time series at the subcooling of 14.81 K (middle subcooling stage) is demonstrated in Fig.3. It reflects the entity attractive feature and local oscillative behavior of the sound pressure attractor.

Fig.3 Pseudo phase diagram of p_1, p_2, p_3 at the subcooling of 14.81 K

(b) Application of wavelet transformation[7,8]

In this work, the fractal feature, say, self-similarity feature of the sound pressure time series, was briefly studied. For example, Fig.4 gives a sound pressure time series of p_2 at the subcooling of 48.95 K and its wavelet transformation series (1000 points) under different scales (here, the Daube-3 wavelet function is adopted). $\log a_n$-$\log T$ relation of a strange peak "A" is plotted in Fig.5, where a_n is the nth scale and T refers to the values of peak "A" at different scales. It is obvious that a certain region of the relation satisfies the power law approximately. Therefore, it could be unrigously concluded that the fractal feature does exist in the sound pressure signals of the subcooled boiling two-phase system at a certain scale range.

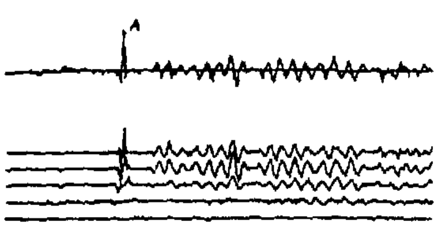

Fig.4 Typical sound pressure time series and its continuous wavelet transformation

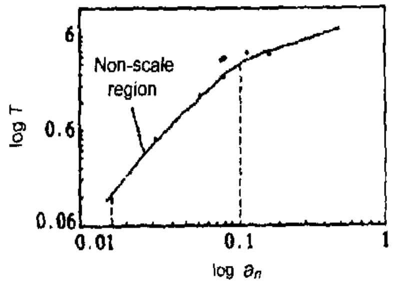

Fig.5 Fractal feature of point "A" in Fig.4 (power law evidence)

3 FRACTAL DIMENSION CALCULATION OF SOUND PRESSURE IN THE SUBCOOLED BOILING SYSTEM

3.1 Fractal dimension and correlation dimension

Since the fractal feature is found in the dynamic characteristics of the subcooled boiling system, it is obvious that the fractal dimension can be applied to describe the dynamic process within the system and it varies with the change of regime. The generalized dimension is defined as follows. The trajectory of an attractor of a dynamic system is divided into a series of points $x(t=0)$, $x(t=\tau)$, $\cdots$, $x(t=N\tau)$, and the d-dimensional phase space is delimited into units of "volume" l^d. The probability of finding a point of attractor within unit i is p_i, and $p_i = \lim_{N\to\infty} N_i/N$, where, N_i is the number of points $(x(t=j\tau))$ in unit i. Thus the generalized dimension is expressed as

$$D_q = \lim_{l\to 0}\frac{1}{q-1}\log[\sum_{i=1}^{M(l)} p_i^q]/\log l \tag{1}$$

Specially, when $q=2$, it is called the correlation dimension, *i.e.*

$$D_2 = -\lim_{l\to 0}\log[\sum_{i=1}^{M(l)} p_i^2]/\log l \tag{2}$$

The D_2 is of great significance for the dynamical characteristics of an ordinary dynamic system, especially when considering the evenness of coverings.[5] For such discrete time series as the acquired sound pressure oscillation signal, G-P algorithm for calculating D_2 directly from time series, which comes from the theory of embedding and reconstruction of the phase space, is devised (see Ref.[9]).

In this work, a computer calculation program was made, which improves the algorithm as well as the following recurrence formula recommended in Ref.[9],

$$r_{i+1,j+1}^{(1)} = r_{i,j}^{(1)} - \mid X_i - X_j \mid + \mid X_{i+m} - X_{j+m} \mid \tag{3}$$

The improvements lie in following aspects. (1) For different embedded dimensions, the recurrence formula

$$r_{i,j,d_0+1}^{(1)} = r_{i,j,d_0}^{(1)} + \mid X_{i+m+1} - X_{j+m+1} \mid \tag{4}$$

is deduced and suggested, where d_0 is the assumed initial embedded dimension. (2) While calculating, the small number series, i. e. the comparing boxes (ε (i)) are arranged from small to big in size. So if $r_{i,j,d_0}^{(1)} > \varepsilon(k)$, then

$$r_{i,j,d_0}^{(1)} > \varepsilon(k+j_1) \quad (j_1 = 1,2,\cdots,l) \tag{5}$$

and the initial dimensions are arranged from low to high. So if $r_{i,j,d_0}^{(1)} > \varepsilon(k)$, then

$$r_{i,j,d_0+n}^{(1)} > \varepsilon(k) \quad (n = 1,2,\cdots,n_1) \tag{6}$$

where n_1 is the assumed highest embedded dimension and l is the assumed maximum comparing box order number.

(3) Finally, some improvements are made in the dynamical array organization. With these arrangements, the calculating efficiency is greatly improved. Using the time series of p_1, p_2, p_3 acquired for an experimental dynamic evolution process, the correlation dimensions at different subcoolings are calculated and demonstrated in Fig.6. The detailed discussion in Ref.[5] indicates that the value of correlation quantitatively reflects some information of dynamic characteristics of the system. It should be pointed out that the G-P algorithm remains to be improved and for the time being, there still exist some errors in the results of calculation, and it may influence the regime recognition to a certain extent.

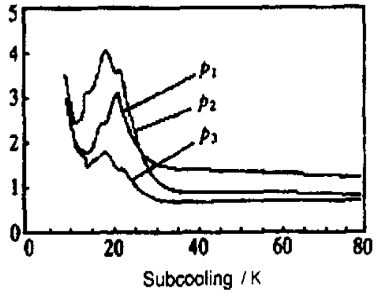

Fig.6 Trend of correlation dimensionof p_1, p_2, p_3

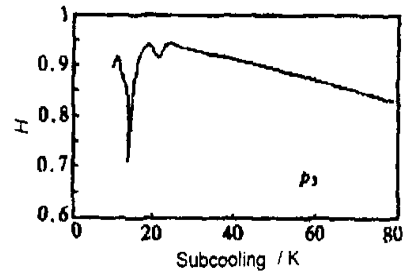

Fig.7 Trend of hurst exponent of p_3

3.2 Hurst exponent

With the coexistence of nonlinearity and randomness of the system process, the acoustical dynamic feature of the subcooled boiling two-phase system is rather complicated. Instead of being complete random, the dynamics shows a certain special irregularity, i.e. the so-called Hurst phenomenon.[10] The analysis of the amplitude distribution diagrams of the sound pressure time series indicates that the sound pressure time series in the subcooled two-phase system is actually not classical Brownian Movement (BM), while its long-term correlation (memory effect) may be identified and explained with Fractal Brownian Movement (FBM) suggested by Mandelbrot *et al.*[11] With the concept of Mandelbrot *et al*'s, Hurst exponent reflects the long-term correlation of the sound pressure time series. When $H>0.5$, the time series is of persistence, *i.e.*, in average, any positive increment in the past implies an increase in the future; and when $H<0.5$, it is of antipersistence, *i.e.*, a positive increment in the past implies a negative one in the future. Therefore, by calculating the value H, the trend of a future time series can be qualitatively forecasted from the persistency.

In fact, Hurst exponent is an excess dimension. Through the R/S analysis method[10] suggested by Hurst, one can evaluate the persistence/antipersistence of the sound pressure time series. The trend of Hurst exponents at typical subcoolings in the experiment (for p_3) is calculated and shown in Fig.7. See Ref.[5] for a detailed discussion to the relation of H and the dynamic feature in the experiment.

3.3 Explanation to the calculation results

For an ideal dynamic system, when $D_2=1$, the system belongs to a self-excited periodic oscillation state (limit cycle); when $D_2=2$, it belongs to a quasi-periodic oscillation; when D_2 is not an integer or $D_2>2$, the system reflects some chaostic feature. In this study, D_2 of p_3 might reflect the dynamic characteristics of the two-phase system because transducer p_3 locates at the a fully developed two-phase region. It is seen from Fig.6 that the variation of D_2 in different stages of the subcoolings may roughly, but comprehensively, demonstrates different dynamic features of the system and its evolution route. When the subcooling >60 K or so (initial stage), $(D_2)_{p_3}\in(0.5, 0.6)$. In this stage, D_2 is relatively small since very small amount of bubbles are generated and stable sound field has not been completely established. When the subcooling is between 60 to 10 K or so (middle), with more and more bubbles appear, a regular sound field is formed, periodic and quasi-periodic pressure oscillations are obviously found. It is natural that $(D_2)_{p_2}\in(0.8, 2.0)$, which means that the two-phase system is self-organized. At low subcooling (near saturated subcooled boiling stage), $(D_2)_{p_3}$ approaches 3.0, which indicates that as more and more bubbles are generated and combined to larger ones, self-organization of the sound pressure oscillation becomes poorer and poorer.

In Fig.7, the trend of H exponent shows that with the decrease of the subcooling, H tends to approach 1.0, which indicates that the system is getting more and more self-organized. However, in the subcooling range of 13 to 14 K, there is an abrupt decrease of H, approaching to 0.5. This phenomenon seems to indicate that there is a kind of "non-equilibrium phase change" and a "critical slowdown" region. That is to say, the pressure oscillation modes on both sides of the critical region are essentially different. Anyhow, it still needs more evidence and further investigation.

Moreover, the H values in the whole subcooled boiling process are all larger than 0.5. This means that the sound pressure time series in the whole subcooled boiling region are persistent.

4 DYNAMIC REGIME RECOGNITION TECHNIQUE BASED ON THE FRACTAL FEATURES

4.1 Regime recognition strategy

The recognition measure given in Fig.8 was adopted in this work, in which, especially, a non-teachered dynamical clustering method was applied to the design of classifier and the strategy of classification. According to the similarity degree of the samples collected in the experiment, an automatic classification and clustering (here, the c-averaged value algorithm was used. See Ref.[5] for detail.) was carried out. The evaluation criterion is the sum of squared error J_c, which is defined as

$$J_c = \sum_{j=1}^{c} \sum_{k=1}^{n_j} \| \vec{V}_k - \vec{m}_j \| \tag{7}$$

where $\vec{m}_j$ (j=1, 2, $\cdots$, c) is the average of samples in the type ω_j, *i.e.*,

$$\vec{m}_j = \frac{1}{n_j} \sum_{j=1}^{n_j} \vec{V}_j \tag{8}$$

and they act as c clustering centers, representing c regime types. n_j is the number of regime samples, $\vec{V}$ is the eigen-vector of each sample and "$\| \cdot \|$" represents a certain distance (in this study, the geometrical distance between eigen-vectors is adopted).

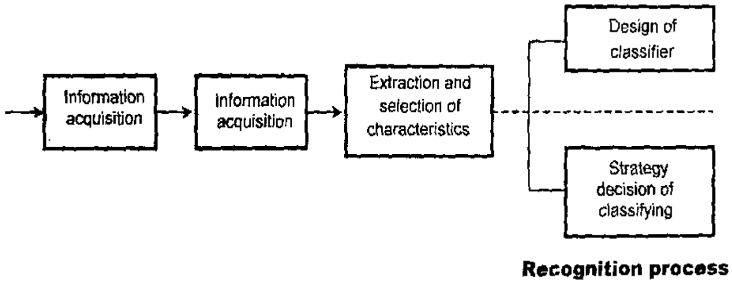

Fig.8 Strategy of regime recognition

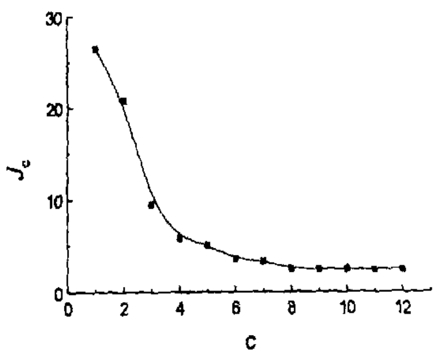

Fig.9 $J_c \sim$ c curve of recognition

The meaning of the criterion is that, the larger the value of J_c is, the worse clustering result is obtained. To find a clustering result satisfying the demand of the smallest J_c which is the aim of the algorithm, the inflexion of $J_c \sim c$ curve which corresponds to the number of recognized regime should be found (as shown schematically in Fig.9). In the work, the results of dynamical clustering based on correlation dimension D_2 and Hurst exponent demonstrate that the number of identified regimes is 3, which is in accordance with the phenomenological analysis to the sound pressure time series.

4.2 Construction of the eigen-vectors for regime recognition based on fractal characteristic quantities

Based on D_2, the eigen-vector for regime recognition is constructed as

$$\vec{V}_{(D_2)_i} = ((\overline{D}_2)_i \overline{\Delta T}_{2i}) \tag{9}$$

where $\overline{(D_2)}_i = (D_2)_i/[(D_2)_i]_{\max}$ and $\overline{\Delta T_{2i}} = \Delta T_{2i}/(\Delta T_{2i})_{\max}$ are normalized correlation dimensions and subcoolings, respectively. Meanwhile, for the eigen-vector based on Hurst exponents, it is defined as

$$\vec{V}_{\mathrm{H}} = (\overline{H}_i, \overline{\Delta T}_{2i}) \tag{10}$$

where $\overline{H}_i = H_i/(H_i)_{\max}$.

4.3 Results of the regime recognition

The results of the subcooled boiling regime recognition based on fractal characteristic quantities of sound pressure waveforms are given in Fig.10 and Fig.11. The results are in rough accordance with the qualitative discussion on the sound pressure time series, their errors occur only at the subcooling range of 30~50 K in the recognition based on D_2. In Ref.[5], it is pointed out that, some improvements in the discrete algorithm of calculating D_2 based on the time series should still be achieved to reduce the error.

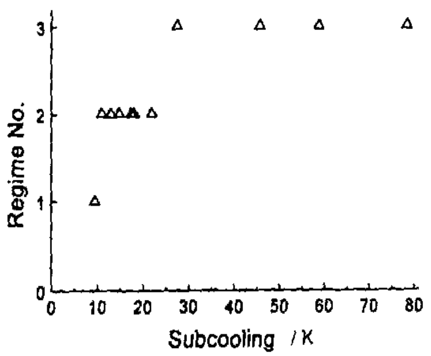

Fig.10 Recognition result based on D_2

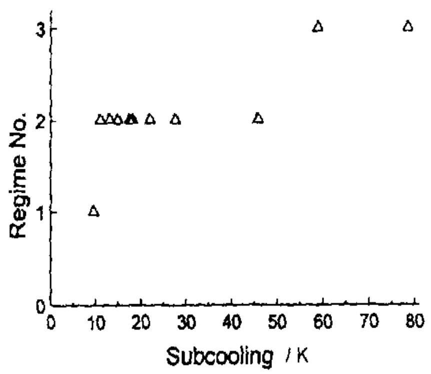

Fig.11 Recognition result based on H value

5 CONCLUSIONS

(1) From the sound pressure data acquired and a through pretreatment procedure to the dynamical pressure signals of the subcooled boiling in tube, combined with phenomenological qualitative analyses, the trend of regime evolution is described.

(2) Based on pseudo the phase space diagram and wavelet analysis, the evidence of the existence of fractal feature in the dynamical subcooled boiling process is demonstratively delivered.

(3) Calculations of the two fractal characteristic quantities-correlation dimension D_2 and Hurst exponent H, for the sound pressure time series aare carried out, which quantitatively reveal the trend of regime evolution in the system.

(4) On the basis of fractal characteristic quantities and through dynamical clustering to them, regimes of the system are recognized. It might contribute to the quantization and intelligent organization of regime recognition in the boiling two-phase system.

(5) Because a great deal of dynamical information is contained in one fractal dimension, the regime identification based on the eigen-vector from the quantity seems rather simple, comprehensive, convenient and hence promising. However, the algorithm of calculating the quantities directly from time series is far from mature and should be improved to reduce the rate of recognition error.

References

1 Saether G. Int J Multiphase Flow, 1990, **17**(4):11~17

2 Franca F, Acikgoz M, Lahey R T Jr *et al.* Int J Multiphase Flow, 1991, **17**(4):545~552

3 Higuchi T. Physica D, 1988, **31**:277~283

4 Djainal D D, Sakuma M, Kozma R *et al.* Characterization of two-phase flow patterns using fractal techniques. In: Proceedings of 2nd International Conference on Multiphase Flow. Kyoto, 1995

5 Kuang Bo. Mechanism and experimental research on subcooled boiling two-phase thermoacoustic system by applying dynamical methods: [Ph. D Thesis], Wuhan: Huazhong University of Science and Technology, 1995

6 Xiang Yu. Experimental and theoretical investigation on sound radiation of vapor bubbles and formation mechanism of self-excited thermoacoustic oscillation in subcooled boiling two-phase flow: [Ph. D Thesis]. Wuhan: Huazhong University of Science and Technology, 1995

7 Jiao Licheng, Bao Zheng. J Electronics (in Chinese), 1993, **21**(7):91~96

8 Qin Qianqin, Yang Zongkai. Practical wavelet analysis (in Chinese). Xi'an: Xi'an University of Electronic Science and Technology Press, 1994

9 Grassberger P, Procaccia I. Characterization of strange attractors. Phys Rev Lett, 1983, **50**(5):346~350

10 Hurst E H. Methods of using long-term storage in reservoirs (I). Proc Instn Civil Engrs, 1956, **5**:519~528

11 Mandelbrot B B, van Ness J W. SIAM Rev, 1968, **10**:422~428