Unified understanding of neutrino oscillation and negative mass-square of neutrino

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Abstract The author indicates that even a conclusive confirmation of neutrino oscillation does not necessarily imply the existence of massive neutrinos. The negative value of neutrino mass-square may be an alternative key with realistic physical meaning. Reexamining special relativity (SR) we find that there actually exists a formal phase velocity of "de Broglie's wave" in temporal Lorentz transformation attributed to the intrinsical essence of Minkowski's space. The properties of spacelike interval between two events have already included constrains to describe superluminal motion and SR is compatible with the faster-than-light motion originally in algebraic domain. Pay attention to that the operator representation, $\vec{p} \leftrightarrow -i\hbar\vec{\nabla}$, has just verified for subluminal particles, not for superluminal particles, adhering to de Broglie's coexistence idea between waves and particles, it is possible to deduce a formal two-component Weyl equation to describe any species of free neutrinos with imaginary rest mass, which is equivalent to making use of the Dirac equation for a free spin-1/2 particle with zero rest mass in form.

Keywords Neutrino oscillation, Superluminal motion, De Broglie's wave, Two-component Weyl equation

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1 INTRODUCTION

The question of whether neutrinos have rest masses is of fundamental importance in physics. It has therefore attracted considerable attention. Since Pontecorvo proposed that there might exist neutrino oscillations phenomenon more than 40 years ago following an analogy with $\kappa^0 \leftrightarrow \bar{\kappa}^0$ oscillations,[1] it is generally agreed that some positive indications of the neutrino oscillation would provide an evidence for neutrino masses and mixings, the deficit puzzles of solar neutrinos as well as atmosphere neutrinos can be understood. At the same time, the existence of neutrino oscillation is therefore a perceptible indication of physics beyond the standard model. In this paper, the author is going to argue: 1) even a conclusive confirmation of the neutrino

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oscillation does not necessarily imply the existence of massive neutrino, 2) neutrinos with pure imaginary "rest mass" may be a realistic physical result, and 3) it is possible to find a unified understanding of the neutrino oscillation and negative mass-square of the neutrino.

2 NEUTRINO OSCILLATIONS

For the discussion of convenience, let us recall the physics of neutrino oscillation phenomenon briefly. If neutrino has mass, a neutrino of definite flavor ν_ℓ created in a charged current interaction along with a corresponding antilepton with "flavor" ℓ (e, μ, τ and perhaps one more) need not be a mass eigenstate, but rather a coherent superposition of mass eigenstates given by

$$|\nu_\ell\rangle = \sum_\alpha U_{\ell\alpha}|\nu_\alpha\rangle \qquad (1)$$

where the ν_α are the mass eigenstates, and the coefficient $U_{\ell\alpha}$ from an unitary matrix U which denotes neutrino mixing. For N species of neutrinos, U is an $N\times N$ matrix. Then a neutrino of one flavor can spontaneously change into one of another flavor as in travelling a distance L (or time t).[1-3] In all practical cases, neutrinos are extremely relativistic, so that the energy of the component ν_α is given by the relativistic energy-momentum relation ($c = \hbar = 1$)

$$E_\ell = \sqrt{\vec{p}^2 + m_\ell^2} \approx |\vec{p}| + \frac{m_\ell^2}{2|\vec{p}|} \qquad (2)$$

where we assume that the 3-momentum $\vec{p}$ of the different flavor are the same, but with different masses. As long as the neutrinos are stable particles, after a time t the evolution of the initial state of Eq.(1) gives

$$|\nu_\ell(t)\rangle = \sum_\alpha e^{-iE_\alpha t}U_{\ell\alpha}|\nu_\alpha\rangle. \qquad (3)$$

The probability of find a $\nu_{\ell'}$ state in the original state ν_ℓ is $P(\nu_\ell \longrightarrow \nu_{\ell'}; L)$, because for extremely relativistic neutrinos $E \approx |\vec{p}|$, making use of Eq.(2) then

$$P(\nu_\ell \longrightarrow \nu_{\ell'}; L) = |\langle\nu_{\ell'}|\nu_\ell(L)\rangle|^2$$

$$= \left|\sum_\alpha e^{-i(M_\alpha^2/2E)}U_{\ell\alpha}U^*_{\ell'\alpha}\right|^2 \qquad (4)$$

Currently, a neutrino oscillation experiment is analyzed for only two neutrino flavors. For instance $\nu_\mu \longrightarrow \nu_e$ mixing, the mixing matrix U takes the form

$$U = \begin{pmatrix} \cos\theta_{\mu e} & \sin\theta_{\mu e} \\ -\sin\theta_{\mu e} & \cos\theta_{\mu e} \end{pmatrix} \qquad (5)$$

Where $\theta_{\mu e}$ is the $\nu_e - \nu_\mu$ mixing angle. Putting matrix (5) into Eq.(4), one may get

$$P(\nu_\ell \longrightarrow \nu_{\ell'}; L) = \sin^2 2\theta_{\mu e}\sin^2(\Delta M^2_{\mu e}L/4E)$$

$$= \sin^2 2\theta_{\mu e} \sin^2(L/L_{\mu e}) \tag{6}$$

where $\Delta M^2_{\mu e} = M^2_\mu - M^2_e$ and the quantity $|L_{\mu e}| = |4E/\Delta M^2_{\mu e}|$ is the oscillation length which gives a distance scale over that the neutrino oscillation effects can be visible.

3 PARAMETERS

Nowadays there are rather compelling evidences for the neutrino oscillation and there are many ongoing ones. To fit the observations one has already obtained the oscillation parameter's ranges of both neutrino mass-squared difference $\Delta M^2_{\beta\alpha}$ and mixing angles $\sin^2 2\theta_{\beta\alpha}$ in an oscillation from one species α to another β.

(a) An evidence obtained from the oscillations of atmospheric neutrino was found in the Super-Kamiokande experiment[4–5] by the hypothesis that $\nu_\mu \longrightarrow \nu_\tau$ oscillation:

$$5 \times 10^{-4}\text{eV}^2 \le \Delta M^2 \le 6 \times 10^{-3}\text{eV}^2, \qquad \sin^2 2\theta > 0.82 \approx 1.$$

(b) For the solar neutrino experiments, by Mikheyev-Smirnov-Wolfenstein (MSW) mechanism to convert a ν_e into a neutrino ν_x of another flavor,[6–8] it follows that

$$5 \times 10^{-6}\text{eV}^2 \le \Delta M^2 \le 10^{-5}\text{eV}^2, \qquad \sin^2 2\theta \approx 7 \times 10^{-3}$$

or in the case of vacuum oscillation. $\Delta M^2 \sim 10^{-10}\text{eV}^2$.

(c) The fitting result of a reported evidence for $\nu_\mu \longrightarrow \bar{\nu}_e$ oscillation from the Los Alamos liquid scintillation neutrino detector (LSND)[9] is

$$0.27\text{eV}^2 \le \Delta M^2 \le 2.3\text{eV}^2.$$

4 NEGATIVE MASS-SQUARE

However, are these different observations and the fitting results the plausible and satisfactory explanation of neutrino being massive? The answer is "no". Because to pay attention to Eq.(6), one may find that the oscillation function (square sine function or cosine function in some papers) is a even function. Hence we can not conclude that $\Delta M^2_{\beta\alpha}$ in the argument of the sine function must be greater than zero. quite the contrary, $\Delta M^2_{\beta\alpha}$ may be negative value, that is,

$$\Delta M^2_{\beta\alpha} \equiv M^2_\beta - M^2_\alpha \le 0 \tag{7}$$

In other words, we seem to be faced with an unavoidable choice, *i.e.*, the mass-square of neutrino may be a negative value in view of mathematical logic, that is both of M^2_β and(or) M^2_α may be negative, therefore even a conclusive confirmation of the neutrino oscillation does not necessarily imply the existence of a nonzero neutrino rest mass.

5 RECENT DATA

The square of the neutrino mass has been measured to be negative in many experiments. Most recent data, which were listed in "Review of Particle Physics, 2000".[10] still include this kind of negative values. For instance, in Refs.[11-12] the weighted average for electro-neutrino ν_e from two experiments reported is

$$m^2(\nu_e) = -0.25 \pm 3.3\text{eV}^2.$$

In the pion decay experiment a negative value for muon-neutrinos was obtained,[13]

$$m^2(\nu_\mu) = -0.016 \pm 0.023\text{MeV}^2.$$

Of course, more accurate results will appear as time goes on and the precision of measurements improves. However, as long as the negative value of the neutrino mass square remains to be obtained, one can not exclude the possibility of them being physical results.

6 PURE IMAGINARY 'REST MASS'

If the negative values of the neutrino mass-square were physical results, they simply mean the "rest mass" of neutrino is a pure imaginary value. May the pure imaginary "rest mass" of neutrino be the true physical mass? An unconventional interpretation "tachyonic neutrino" was given 25 years ago[14-15]. Unfortunately, the driving force for research on this issue has been lacking in view of: a) besides negative mass-square there was no piece of experimental evidence to support tachyonic conjecture strongly; b) as yet there does not exist a completely satisfactory quantum field theory of any type of tachyon.[16] Indeed, it is still a question whether the pure imaginary "rest mass" of neutrino may lead to a two-component quantum field to guarantee that a neutrino always has left-handed spin and an anti-neutrino always has right-handed spin.

7 TEMPORAL LORENTZ TRANSFORMATION

After the publication of Einstein's original paper on special relativity (SR), it seems clear that no object, influence, or interaction (force) can move or be transmitted faster than the speed of light in vacuum. In other word, it is generally agreed that according to SR the speed of light in vacuum is the maximum speed of propagation of signals in nature. However, reexamine temporal Lorentz transformation

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{t - \frac{x}{\frac{c^2}{v}}}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{8}$$

one may find that there exists a superluminal speed in temporal Lorentz transformation, i.e. c^2/v is the like-phase velocity of "de Broglie's wave" related to v. It is inconceivable that this point has been overlooked for over 70 years.

8 SUPERLUMINAL MOTION

In Minkowski's space the interval between two events is an invariant for the proper Lorentz transformations (i.e. Lorentz transformations without space inversion and time inversion),

$$s^2 = x^2 + y^2 + z^2 - c^2t^2 = x'^2 + y'^2 + z'^2 - c^2t'^2 \tag{9}$$

This invariant may be positive (spacelike), zero (light cone), or negative (timelike), so that the following inferences must be drawn.[17]

a) The particles will keep their essential qualities(tachyon, photon or luxon, bradyon) in their born time for the proper Lorentz transformations. In other words, only by means of accelerating or decelerating technique nobody can make the particles moving beyond the light barrier.

b) There do not exist the superluminal or light inertial frames in nature, hence there remain no problems relating proper length and proper time to be solved. Only in the subluminal inertial frames or in timelike space we can discuss the superluminal or light motion. Therefore, even concerning the superluminal and light motion, the relative velocity of the used inertial frames is always less than the speed of light. Three situations in Eq.(9) are just described in timelike space.

c) As long as not to refuse imaginary number, almost all the formula in SR need not to alter to be applicable to the three situations in Eq.(9). For instance, for a neutral bradyon the energy-momentum relation in SR is

$$E^2 = c^2p^2 + m_0^2c^4 \tag{10}$$

Recall the studies of tachyon,[18-19] introducing imaginary number "i" as well as a real meta-mass m^* to denote the imaginary rest mass m_0 of a tachyon with speed $\vec{u}$, then we have

$$m_0 = im^* \tag{11}$$

$$E^2 = c^2p^2 - m^{*2}c^4 \tag{12}$$

where

$$E = \frac{m^* c^2}{\sqrt{\frac{u^2}{c^2} - 1}}, \qquad |\vec{p}| = \frac{m^* u}{\sqrt{\frac{u^2}{c^2} - 1}}. \tag{13}$$

Hence SR is compatible with faster-than-light motion originally in algebraic domain.

d) Superluminal motion could unlikely confuse the problem of causality. Making use of the temporal Lorentz transformation one may obtain the time interval $(t'_2 - t'_1)$ between the events as viewed $(t_2 - t_1)$ in some other frame moving with velocity v would be given by $(t'_2 - t'_1) = \gamma[(t_2 - t_1) - v(x_2 - x_1)/c^2]$, where $\gamma = (1 - v^2/c^2)^{-1/2}$. Thus if $(t_2 - t_1)$ and $(t'_2 - t'_1)$ were of opposite sign, the condition must be

$$(t_2 - t_1) - \frac{v(x_2 - x_1)}{c^2} < 0,$$

$$\frac{(x_2 - x_1)}{(t_2 - t_1)} > \frac{c^2}{v}. \tag{14}$$

It is obvious that even according to SR in the range of $(c, c^2/v)$ the superluminal motion could not mix up the causality. For example 1, the earth's orbit velocity is ~ 30 km/s, then $c^2/v \sim 10000c$, that is if some superluminal phenomenon with velocity $<$ $10000c$ happen on the sun, an observer on the earth would not confuse the problem of cause and effect. For example 2, an observer does experiments in laboratory on the earth, the velocity between he/she and the instruments is ~ 0 km/sec, the $c^2/v \sim \infty$, that is any superluminal phenomena occuring during those experiments would not mix up the question of causality. Therefore, the maximum speed of propagation of signals between two inertial frames in nature is c^2/v being related to their relative velocity v. It is thus clear again that relativity is absolutely relative![20]

9 HAMILTONIAN

It is well known that, generally, the Hamiltonian function H for a particle in a conservative field is equal to the total energy, so that Schrödinger's discovery in 1925 of a nonrelativistic wave equation led immediately to the suggestion, by Schrödinger and by a number of other physicists, that the wave equations for any kinds of particle such as Pauli, Klein-Gordon and Dirac equations could be obtained by making the operator representation substitutions ,

$$\vec{p} \leftrightarrow -i\hbar\vec{\nabla}, \qquad E \leftrightarrow i\hbar\frac{\partial}{\partial t} \tag{15}$$

into the corresponding Hamiltonian function H in the following equation,

$$\hat{H}\Psi = E\Psi. \tag{16}$$

However, the above correspondences have just verified for subluminal particle. We are really not sure whether the above operator representation substitutions may be used for superluminal particles. The present author thinks that the erroneous usage of Eq.(15) for superluminal momentum is the failure reason to quest for a satisfactory quantum field theory of any superluminal particle in the recent 40 years. How to construct a quantum field theory of tachyon in timelike frames correctly? In order to solve this issue we may draw some inspiration from the de Broglie's idea. The de Broglie's personal view on "matter waves" appeared in his famous first three papers in the Comptes Rendus de l'Acadèmie des Sciences in 1923 about his study on the law of harmony of phases[21]. In fact, de Broglie's main idea is not dualism, but coexistence between waves and particles, i.e. we must assume the existence of particles always accompanied by waves. Follow de Broglie's idea, we may use a dual subluminal speed $\vec{v}$ to replace a superluminal speed $\vec{u}$ in terms of following relation

$$u = \frac{c^2}{v}. \tag{17}$$

Eq.(13) turns to

$$p = \frac{m^* u}{\sqrt{\frac{u^2}{c^2} - 1}} = \frac{m^* \frac{c^2}{v}}{\sqrt{\frac{c^4}{v^2 c^2} - 1}} = \frac{m^* c}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{m^* v}{\sqrt{1 - \frac{v^2}{c^2}}} \frac{c}{v} = p^* \frac{c}{v}$$

$$E = \frac{m^* c^2}{\sqrt{\frac{u^2}{c^2} - 1}} = \frac{m^* c^2}{\sqrt{\frac{c^4}{v^2 c^2} - 1}} = \frac{m^* vc}{\sqrt{1 - \frac{v^2}{c^2}}} = p^* c. \tag{18}$$

The Hamiltonian operator $\widehat{H}$ for a free spin-1/2 particle with zero mass is

$$\widehat{H} = \sqrt{\widehat{\vec{p}}^{*2} c^2 + m_0^2 c^4}|_{m_0=0} = (c\vec{\alpha} \cdot \widehat{\vec{p}}^* + \beta m_0 c^2)|_{m_0=0} = -i\hbar c\vec{\alpha} \cdot \vec{\nabla} \tag{19}$$

where $\vec{\alpha}$, for a subluminal particles, is the Dirac matrices and each of the Dirac matrices is anti-commuting Hermitean. From Eq.(18) for a free spin-1/2 tachyon, its corresponding Hamiltonian operator is just the same as in Eq.(19). It is unnecessary to go into detail, starting from Eq.(19) and carrying on with some work, we must deduce the a familiar formal two-component Weyl equation for describing neutrinos, which is related to the maximum parity violation discovered in 1956 by Lee and Yang[22-23]. Of course, the relativistic invariance of the two-component Weyl equation for proper Lorentz transformations is well known and corresponding quantum field theory, which may guarantee a neutrino always with left-handed spin and an anti-neutrino always with right-handed spin, is satisfactory.

10 SUMMARY

This paper has given an unified understanding of the neutrino oscillation and negative mass-square of neutrino. 1) Even a conclusive confirmation of the neutrino oscillation does not necessarily imply the existence of massive neutrino. 2) The existence of neutrinos with negative mass-square may be an alternative key to realistic physical meaning. 3) Neutrino may be tachyon. 4)Introducing imaginary number "i" as well as a real meta-mass m^* to denote the imaginary proper mass m_0 for a superluminal particle, without additional postulates special relativity is compatible with superluminal motion. 5) As long as to follow de Broglie's coexistence idea and to use the superluminal momentum operator representation $\vec{p} \leftrightarrow -i\hbar\vec{\nabla}/\vec{\alpha}$ (from Eq.(18)), where $c\vec{\alpha}$ is the operator of dual subluminal velocity $\vec{v}$, it is possible to deduce a formal two-component Weyl equation to describe any species of neutrinos with imaginary "rest mass", which is equivalent to making use of the familiar Dirac equation for a free spin-1/2 particle with zero mass, just as Lee and Yang did in 1956.

References

1 Pontecorvo B. Sov Phys JETP, 1968, **26**: 984

2 Gribov V, Pontecorvo B. Phys Lett, 1969, **B28**: 493

3 Bilenky S, Pontecorvo B. Phys Reports, 1978, **C41**: 225

4 Hirata K S *et al.* Phys Lett, 1992, **B280**:146

5 Fukuda Y *et al.* Phys Rev Lett, 1998, **81**:1562

6 Wolfenstein L. Phys Rev, 1978, **D17**:2369

7 Wolfenstein L. Phys Rev, 1979, **D20**:2634

8 Krastev P, Petcov S. Phys Rev, 1996, **D53**:1665

9 Caldwell D, Mohapatra R. Phys Rev, 1993, **D48**:3259

10 Review of Particle Physics, Euro Phys J, 2000, **C15**:344

11 Weinhermer C *et al.* Phys Lett, 1999, **B460**:219

12 Lobashev V *et al.* Phys Lett, 1999, **B460**:227

13 Assamagan K *et al.* Phys Rev, 1996, **D53**:6065

14 Chang Tsao. Proceedings of the Sir A. Eddington Century Symposium (Singapore: World Scientific Publishing Co.), Vol.3, 1986. 431

15 Chodos A *et al.* Phys Lett, 1992. **B150**:431

16 Kamoi K, Kamefuchi S. Prog Theor Phys, 1971, **45**:1646

17 AI X B (Ai,H.B.), J Yunnan Univ of the Nationalities, 2000, **9**(4):194

18 Bilaniuk O, Deshpande V, Sudarshan E. Am J Phys, 1962, **30**:718

19 Feinberg G. Phys Rev, 1967, **159**:1089

20 Ai H B. Phys Essays, 1998, **11**:1

21 de Broglie L. Thesis, Paris, 1924. Ann Physik, 1925, **3**:22

22 Lee T D, Yang C N. Phys Rev, 1956, **104**:254

23 Lee T D, Yang C N. Phys Rev, 1957,**105**:1671