# EFFECT OF HEATING RATE ON NONLINEARITY OF DOSE RESPONSE IN TL BY KINETICS MODEL*

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**ABSTRACT**

The general-order kinetics model of thermoluminescence is used to study the effect of heating rate on the nonlinearity of dose response in thermoluminescence. The results show that when we consider the dose dependence of the maximum intensity of the glow peak, the effect of the heating process on dose response is supralinear when the order of kinetics $b>1$, and this effect is enhanced with the increase in heating rate.

**Keywords** Thermoluminescence, Heating rate, Kinetics model, Dose response

## 1 INTRODUCTION

When considering the dose dependence of the area under the glow curve of thermoluminescence(TL), it is well known that the dose response function $f(D)$ of TL is highly dependent on both the ionization density, i.e. the linear energy transfer (LEF) of the incident radiation, and the nature of the host material. Moreover, when we consider the dose denpendence of the maximum intensity of glow curve $I_{\mathrm{m}}$, due to that the shape of glow curve depends on the heating process, $f(D)$ depends also on the heating process. There are experimental results which show that the nonlinearity of $f(D)$ is increased with the heating rate[1,2]. Using the second-order kinetics model, one can expect that $I_{\mathrm{m}}$ varies with the dose $D$ as $D^p$ where $p$ slightly exceeds unity $(1.05\leq p\leq 1.10)$[3], which shows immediately that $I_{\mathrm{m}}$ varies slightly supralinearly with the dose $D$, but the effect of heating rate on the dose response is not given. In this paper, the general-order kinetics model of TL will be used to study this effect theoretically.

## 2 MATHEMATICS ANALYSIS AND RESULTS

For the convenience of analysing the effects of the heating rate on the dose response, the dose response function $f(D)$ is defined as

$$f(D) = [\mathrm{d}I_{\mathrm{m}}/\mathrm{d}D]/[\mathrm{d}I_{\mathrm{mL}}/\mathrm{d}D_{\mathrm{L}}] \qquad (1)$$

where $I_{\mathrm{mL}}$ is the maximum TL intensity at a refering dose $D_{\mathrm{L}}$ which is in low dose range. This dose response function $f(D)$ can be devided into two parts as

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$$f(D) = f_s(D) \cdot f_h(n_0) \tag{2}$$

where

$$f_s(D) = [\mathrm{d}n_o/\mathrm{d}D]/[\mathrm{d}n_{oL}/\mathrm{d}D_L] \quad \text{and} \quad f_h(n_0) = [\mathrm{d}I_m/\mathrm{d}n_o]/[\mathrm{d}I_{mL}/\mathrm{d}n_{0L}] \tag{3}$$

where $n_0$ is the initial concentration of trapped carriers and when dose is equal to $D_L$, we denote it by $n_{0L}$. The $f_s(D)$ which is related to the dose dependence of the initial concentration of trapped carriers does not depend on the heating rate, so it will not be discussed in this paper. We will focus our attention to the $f_h(n_0)$ which is related to the heating rate.

The glow intensity for a general-order kinetics for a linear heating rate is given by[4]

$$I = sn_0^b \exp(-E/KT)\{\frac{(b-1)}{\beta} sn_0^{b-1} F(T) + 1\}^{-b/(b-1)} \tag{4}$$

and the maximum condition is

$$\frac{(b-1)sn_0^{b-1}}{\beta} F(T_m) + 1 = \frac{sn_0^{b-1} bKT_m^2}{\beta E} \exp(-E/KT_m) \tag{5}$$

where

$$F(T) = \int_{T_0}^{T} \exp(-E/KT')\mathrm{d}T'$$

$s$ is a pre-exponential factor, $E$ is the activation energy, $K$ is Boltzmann's constant, $T_m$ is the temperature at the maximum, $\beta$ is the heating rate and $b$ is the order of kinetics. It is well known that $F(T)$ can be replaced by an asymptotic series[5]

$$F(T) = T\exp(-E/KT) \sum_{n=1} (-1)^{n-1} n! (KT/E)^n \tag{6}$$

Taking derivatives of equation(5) with respect to $n_0$ and $\beta$ respectively and expanding them into power series of $KT_m/E$, one can get the expressions

$$\mathrm{d}T_m/\mathrm{d}\beta = [KT_m^2/E\beta]\left\{1 - KT_m/E + (6-2b)(KT_m/E)^2 + \ldots\right\} \tag{7}$$

and

$$\frac{\mathrm{d}T_m}{\mathrm{d}n_0} = -(b-1)\frac{KT_m^2}{En_0}\left\{1 - \frac{KT_m}{E} + (6-2b)(\frac{KT_m}{E})^2 + \ldots\right\} \tag{8}$$

Using equations (4), (5), (6) and (8), one can get

$$\mathrm{d}I_m/\mathrm{d}n_0 = [C\beta E/KT_m^2]\left\{1 + 2bKT_m/E - 4(KT_m/E)^2 + \ldots\right\} \tag{9}$$

where $C$ is a constant.

Inserting Eq.(9) to Eq.(3), and expanding it into power series of $KT_m/E$ and $KT_{mL}/E$, one has

$$f_h(n_0) = \frac{T_{\rm mL}^2}{T_{\rm m}^2}\{1 + 2b\frac{K}{E}(T_{\rm m} - T_{\rm mL}) + 4\left(\frac{K}{E}\right)^2(T_{\rm mL}^2 - T_{\rm m}^2) + 4b^2\left(\frac{KT_{\rm mL}}{E}\right)^2 + \ldots\} \quad (10)$$

where $T_{\rm mL}$ is the temperature at the maximum refering to the dose $D_{\rm L}$. Noting that $E \gg KT_{\rm m}$ in general, the value of $E$ is typically around $20KT_{\rm m}$, by only keeping the first term of equations, we can derive approximately

$$\mathrm{d}T_{\rm m}/\mathrm{d}\beta = KT_{\rm m}^2/E\beta \quad (11)$$

$$\mathrm{d}T_{\rm m}/\mathrm{d}n_0 = -(b-1)KT_{\rm m}^2/En_0 \quad (12)$$

$$f_h(n_0) = T_{\rm mL}^2/T_{\rm m}^2 \quad (13)$$

and

$$\mathrm{d}f_{\rm h}(n_0)/\mathrm{d}\beta = (2KT_{\rm mL}^2/\beta ET_{\rm m}^2)(T_{\rm mL} - T_{\rm m}) \quad (14)$$

## 3 DISCUSSIONS

In the present work, the general-order kinetics model is used to theoretically study the effects of the heating rate on the nonlinearity of dose response in TL. The expression of the dose response to the heating rate is deduced based on the general-order kinetics. From Eqs.(11) and (12), one can see that the temperature $T_{\rm m}$ increases with the heating rate $\beta$ and the change of $T_{\rm m}$ with the initial concentration of trapped carriers $n_0$ is dependent on the order of kinetics $b$. For $b>1$, $T_{\rm m}$ decreases with increasing $n_0$; for $b<1$, $T_{\rm m}$ increases with $n_0$; and for $b=1$, $T_{\rm m}$ will not change. Equation (13) shows that $f_{\rm h}(n_0)$, which is the part of the dose response function relating to the heating rate, is inversely proportional to $T_{\rm m}^2$. Thus for $b>1$, $b<1$ and $b=1$, $f_h(n_0)$ is supralinear, sublinear, and equal to 1 respectively according to the kinetics model. It can also be seen from Eq.(14) that these effects are more obvious for larger heating rates.

In general, the order of kinetics $b$ is assumed to be larger than 1. In this case, from the above discussions, one comes to the conclusion that the effect of heating rate on the dose response is supralinear, and this effect is enhanced with the increase in heating rate, which is in agreement with experimental results.

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