Parity violation and neutrino mass

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Abstract   Besides the fact of parity violation in weak interactions, based on evidences from neutrino oscillation and tritium beta decay, a natural conjecture is that neutrinos may be spacelike particles with a tiny proper mass. A Dirac-type equation for spacelike neutrinos is further investigated and its solutions are discussed. This equation can be written in two spinor equations coupled together via nonzero proper mass while respecting maximum parity violation.

Keywords   Parity violation, Spacelike neutrinos, Nonzero proper mass

CLC numbers   O413.1, O412, O572.32^{+}1   A

1 INTRODUCTION

Parity violation is a specific feature of the weak interactions. It was first discovered by Lee and Yang in 1956[1] and experimentally established by Wu in beta-transition of polarized Cobalt nuclei.[2] In the standard model, neutrinos are massless. Three flavors of neutrinos are purely left-handed, but anti-neutrinos are right-handed. In recent years, many evidences for neutrino oscillation come from the solar and atmospheric neutrino data have shown that neutrinos have tiny mass (about 1 eV) or mass difference.[3-5]

If neutrino has a tiny rest mass, it would move slower than light. When taking a Lorentz boost with a speed faster than the neutrino, the helicity of the neutrino would change its sign in the new reference frame. In another word, parity would not be violated in weak interactions. In order to solve this dilemma, the hypothesis that neutrinos may be spacelike particles is further investigated in this paper.

Besides neutrino oscillations, recent measurements in tritium beta decay experiments have presented a value of negative mass squared, $m^2(\nu_e) \approx -2.5\,\mathrm{eV}^2$.[6-8] Moreover, the muon neutrino also exhibits a negative mass squared.[9] These results suggest that neutrinos may be tachyons.

The negative value of neutrino mass-square simply means:

$$E^2/c^2 - p^2 = m_\nu^2 c^2 < 0 \tag{1}$$

The right-hand side in Eq.(1) can be rewritten as $(-m_s^2 c^2)$, then m_s has a positive value. m_s is called the proper mass or "meta mass" since it is different from the rest mass. The subscript s means spacelike particle, i.e. tachyon. The negative value on the righthand side of Eq.(1) means that p^2 is greater than $(E/c)^2$.[10-12]

A Dirac-type equation for tachyons was investigated by Chodos et $al.$[13] A form of the lagrangian density for tachyonic neutrinos was proposed. More theoretical work can be found in Ref.[14-16]. In a recent paper, Ai has given a unified consideration for neutrino oscillation and negative mass-square of neutrinos, which should be paid more attention.[17]

2 THEORY

To follow Dirac's approach,[18] the Hamiltonian form of spacelike Dirac-type equation for neutrinos can be written in:

$$\hat{E}\Psi = c(\vec{\alpha} \cdot \hat{p})\Psi + \beta_s m_s c^2 \Psi \tag{2}$$

where $\hat{E} = i\hbar\partial/\partial t, \hat{p} = -i\hbar\nabla$. $\vec{\alpha} = (\alpha_1, \alpha_2, \alpha_3)$ and β_s is 4×4 matrix, which are defined as

$$\alpha_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}, \quad \beta_s = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}, \tag{3}$$

where σ_i is 2×2 Pauli matrix, I is 2×2 unit matrix. The commutation relations are as follows:

$$\alpha_i\alpha_j + \alpha_j\alpha_i = 2\delta_{ij},$$

$$\alpha_i\beta_s + \beta_s\alpha_i = 0,$$

$$\beta_s^2 = -1, \tag{4}$$

where $\beta_s = \beta\gamma_5$. The spacelike Dirac-type equation (2) can be rewritten in covariant forms[19] by multiplying matrics β and γ_5. Denote the bispinor function Ψ by two spinor functions: φ and χ, then Eq.(2) can be rewritten as a pair of two-component equations:

$$i\hbar\frac{\partial\varphi}{\partial t} = -ic\hbar\vec{\sigma} \cdot \nabla\chi + m_s c^2\chi,$$

$$i\hbar\frac{\partial\chi}{\partial t} = -ic\hbar\vec{\sigma} \cdot \nabla\varphi - m_s c^2\varphi. \tag{5}$$

From Eq.(5), the conserved current can be derived:

$$\frac{\partial\rho}{\partial t} + \nabla \cdot \vec{j} = 0 \tag{6}$$

and we obtain

$$\rho = \Psi^\dagger \gamma_5 \Psi, \quad \vec{j} = c(\Psi^\dagger \gamma_5 \vec{\alpha} \Psi), \tag{7}$$

where ρ and $\vec{j}$ are probability density and current; $\Psi^\dagger$ is the Hermitian adjoint of Ψ.

Considering a plane wave along the z axis for a right-handed particle, $\bar{\nu}$, the helicity $H = (\vec{\sigma} \cdot \vec{p})/|\vec{p}| = 1$, then Eq.(5) yields the following relation:

$$\chi = \frac{cp - m_s c^2}{E} \varphi. \tag{8}$$

When taking massless limitation, $m_s = 0$, we obtain $E = cp$ and $\chi = \varphi$. Then Eq.(5) should be decoupled and reduced to two-component Weyl equation.[15,16]

The plane wave can be represented by $\Psi(z,t) = \psi_\sigma exp[\frac{i}{\hbar}(pz - Et)]$, where ψ_σ is a four-component bispinor. Substituting this bispinor into the wave equation (2) or (5), the explicit form of two bispinors with positive-energy states are listed as follows:

$$\psi_1 = \psi_{\uparrow(+)} = N \begin{pmatrix} 1 \\ 0 \\ A \\ 0 \end{pmatrix}, \quad \psi_2 = \psi_{\downarrow(+)} = N \begin{pmatrix} 0 \\ -A \\ 0 \\ 1 \end{pmatrix} \tag{9}$$

and other two bispinors with the negative-energy states are:

$$\psi_3 = \psi_{\uparrow(-)} = N \begin{pmatrix} 1 \\ 0 \\ -A \\ 0 \end{pmatrix}, \quad \psi_4 = \psi_{\downarrow(-)} = N \begin{pmatrix} 0 \\ A \\ 0 \\ 1 \end{pmatrix}, \tag{10}$$

where the component A and the normalization factor N are chosen as

$$A = \frac{cp - m_s c^2}{|E|}, \quad N = \sqrt{\frac{p + m_s c}{2 m_s c}}. \tag{11}$$

For $\psi_1 = \psi_{\uparrow(+)}$, the conserved current in Eq.(6) becomes

$$\rho = \Psi_1^\dagger \gamma_5 \Psi_1 = \frac{|E|}{m_s c^2}, \quad j = \frac{p}{m_s}. \tag{12}$$

Clearly, the ratio j/ρ represents the superluminal speed u_s. For $\psi_2 = \psi_{\downarrow(+)}$, the density ρ is negative so that it should be discarded. If we consider the negative states as mathematics solutions in the preferred frame, then $\psi_1 = \psi_{\uparrow(+)}$ is the only solution with physical identity. It gives a natural choice that antineutrino is right-handed only. If we identify the preferred frame with Cosmic Microwave Background Radiation (CMBR), the earth frame can be considered as the preferred frame approximately ($v/c \approx 10^{-3}$). Further study on the negative states and negative ρ may be associated with complicated mathematics under Generalized Galilean Transformation (GGT) (see Ref.[14]),

which will not be discussed here. In addition, the pseudo scalar for each spinor satisfies:
$\bar{\Psi}\gamma_5\Psi = \Psi^+\gamma^o\gamma_5\Psi = 0$.

3 DISCUSSION

It has been shown that the spacelike Dirac-type equation (2) reduces to the two component Weyl equation in the massless limit.[15,16] Notice that Eq.(2) is valid for the right-handed antineutrinos. In order to describe the left-handed neutrino, we now take a minus sign for the momentum operator, then Eq.(2) becomes

$$\hat{E}\Psi_\nu = -c(\vec{\alpha} \cdot \hat{p})\Psi_\nu + \beta_s m_s c^2\Psi_\nu \qquad (13)$$

Similar to the solutions for Eq.(2), Eq.(13) yields one physical solution for the neutrino: $\psi_\nu = \psi_{\downarrow(+)}$. Therefore, only $\bar{\nu}_R$ and ν_L exist in nature.

As is shown in Ref.[12,19] the energy of a tachyonic neutrino (or anti-neutrino), E_ν, could be negative in non-preferred reference frames. With respect to CMBR we obtain $\Delta E_\infty \approx 10^{-3}$ eV in the earth frame since the electron neutrino mass is about 1 eV. ΔE_x is a undetectable effect at present time.

The electron neutrino and the muon neutrino may have slightly different proper masses. It provides a natural explanation why the numbers of e-lepton and μ-lepton are conserved respectively at least for low energy experiments.

Comparing with the electron mass, the mass term of the e-neutrino in Eq.(2) is approximately close to zero. Moreover, from the result $\bar{\Psi}\gamma_5\Psi = 0$ for Spacelike neutrinos, it means that the mass term in Eq.(2) may be negligible in most experiments. Therefore, spacelike neutrinos behave just like the massless neutrinos. This similarity may also play role at the level of SU(2) gauge theory. Indeed, the hypothesis of spacelike neutrinos is a development of the two-component model of the massless neutrino, in which the reversal of its helicity is completely impossible. On the other hand, if the neutrino has a small rest mass, it would be against the fact of parity violation in weak interactions.

According to special relativity[20], if there is a spacelike particle, it might travel backward in time. Besides the re-interpretation rule introduced in the 1960's,[10,11], another approach is to introduce a kinematic time under GGT, which always goes forward.[21-23] Therefore, special relativity can be extended to the spacelike region without causality violation.

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