Critical phenomena of a nuclear system in alternative ZM models

SONG Hong-Qiu[1,2]

([1]*Research Center of Nuclear Theory of National Laboratory of Heavy Ion Accelerator of Lanzhou, Lanzhou 730000;*

[2]*Shanghai Institute of Nuclear Research, the Chinese Academy of Sciences, Shanghai 201800*)

Abstract Alternative versions of the ZM model are extended to asymmetric nuclear matter by including ρ meson degree of freedom in the Lagrangian. The extended models are then used to study the thermodynamical properties of asymmetric nuclear matter at finite temperature. The critical temperature for a liquid- gas phase transition in nuclear matter and its dependence on asymmetry parameter are calculated. The limiting temperature T_{lim}, which reflects Coulomb instability of hot nuclei, is studied. The calculated results are compared with that given by the original ZM model.

Keywords Critical phenomena, Nuclear matter, ZM model

CLC numbers O572.33, O571.42+5

1 INTRODUCTION

The interest of studying the liquid-gas phase transition in nuclear matter[1] has increased recently with the attempt by the EOS Collaboration to extract critical exponents of fragmenting nuclear systems produced in the collision of 1 GeV/nucleon Au nuclei with a carbon target[2] and with the extraction by the ALADIN/LAND Collaboration of a caloric curve resulting from the fragmentation of the quasiprojectile formed in the collision Au+Au at 600 MeV/nucleon exhibiting a behavior expected for a first-order liquid-gas phase transition.[3] Since the system formed in the collision is of finite size and with the Coulomb interaction, its limiting temperature T_{lim} is also important.[4] Below T_{lim}, the nucleus can exist in equilibrium with the surrounding vapor. But above T_{lim}, the nucleus is unstable and will fragment. This is the so-called Coulomb instability of hot nuclei.

Although the Walecka model[5] is successful in describing the properties of both infinite nuclear matter and finite nuclei, it gives too large a compression modulus (540 MeV). Recently, there have been various prescriptions for modifying the Walecka model, such as the nonlinear sigma-omega model[6], the scalar derivative coupling model[7], the quark-meson coupling model[8], etc. In the ZM model, a derivative coupling between baryon and

Supported in part by National Natural Science Foundation of China under (19675060) and Chinese cademy of Sciences under contract (KJ951-A1-410)

Manuscript received date: 2000–01–04

scalar meson is introduced. It yields much more reasonable value of 225 MeV for the compression modulus. The ZM model has also been applied to discuss the liquid-gas phase transition of nuclear matter and the Coulomb instability of hot nuclei.[9] Recently, variants of the ZM model were implemented and applied to dense cold[10] as well as warm[11] nuclear matter. In these new versions of the ZM model, a nonlinear coupling between the nucleon and the vector ω meson is introduced. A recent finite nuclei calculation[12] shows that the modified version, ZM3 model, improves upon the original ZM model regarding the energy splitting of levels due to the spin-orbit interaction in finite nuclei. It is therefore of interest to apply these new versions in studying the critical phenomena of a nuclear system and compare the results with those given by the original ZM model.

The paper is organized as follows. In section 2, we will introduce the equation of state (EOS) for bulk nuclear matter at finite temperature in the ZM models and a two phase equilibrium model for the study of the Coulomb instability of hot nuclei. Section 3 contains the results and some discussions.

2 MODEL

After rescaling the nucleon field as $\psi \rightarrow m^{*1/2}\psi$ for all ZM models and rescaling the vector meson field as $\omega_\mu \rightarrow m^*\omega_\mu$ for ZM2 and ZM3 models, the Lagrangian in the ZM models reads

$$\begin{aligned}\mathcal{L} = \; & \bar{\psi} i\gamma_\mu\partial^\mu\psi + m^{*\alpha}(-g_\omega\bar{\psi}\gamma_\mu\psi\omega^\mu - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}m_\omega^2\omega_\mu\omega^\mu) \\ & -\bar{\psi}g_\rho\tfrac{1}{2}\gamma_\mu\vec{\tau}\cdot\vec{b}^\mu\psi - \bar{\psi}(M - m^{*\beta}g_\sigma\sigma)\psi \\ & +\tfrac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) - \tfrac{1}{4}\vec{L}_{\mu\nu}\cdot\vec{L}^{\mu\nu} + \tfrac{1}{2}m_\rho^2\vec{b}_\mu\cdot\vec{b}^\mu \end{aligned} \tag{1}$$

In the above equation, M is the rest mass of nucleon and $\vec{\tau}$ is the Pauli operator. σ, ω_μ and $\vec{b}_\mu$ are the neutral scalar meson field, neutral vector meson field and isovector vector meson field, with parameters m_σ, g_σ, m_ω, g_ω, m_ρ and g_ρ , respectively. The effective mass M^* is defined as $M^* = m^*M$ with $m^* = [1 + g_\sigma\sigma/M]^{-1}$. $F_{\mu\nu} \equiv \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$, $\vec{L}_{\mu\nu} \equiv \partial_\mu\vec{b}_\nu - \partial_\nu\vec{b}_\mu$ and α and β have the following values for different models:for ZM, $\alpha = 0$, $\beta = 1$; ZM2, $\alpha = 1$, $\beta = 1$; for ZM2, $\alpha = 2$, $\beta = 1$.

In a mean-field approximation(MFA), the meson fields are replaced by their mean field values, σ_0, ω_0 and b_0. By using the standard technique in the field theory, we obtained the expression for the pressure of the system as

$$\begin{aligned} p \; = \; & \frac{C_\omega^2}{2M^2}m^{*\alpha}\rho^2 - \frac{M^4}{2C_\sigma^2}(\frac{1-m^*}{m^{*\beta}})^2 + \frac{C_\rho^2}{8M^2}\rho_3^2 \\ & + \sum_{q=n,p}\frac{2}{3(2\pi)^3}\int d^3k\frac{\vec{k}^2}{E_k^*}[n_q(k) + \bar{n}_q(k)] \end{aligned} \tag{2}$$

In Eq.(2), $E_k^* = \sqrt{\vec{k}^2 + M^{*2}}$ and we introduced $C_\sigma^2 = g_\sigma^2M^2/m_\sigma^2$, $C_\omega^2 = g_\omega^2M^2/m_\omega^2$ and $C_\rho^2 = g_\rho^2M^2/m_\rho^2$. Fermi distributions are defined as $n_q(k) = \{\exp[(E_k^* - \nu_q)/k_BT] + 1\}^{-1}$

and $\bar{n}_q(k) = \{\exp[(E_k^* + \nu_q)/k_BT] + 1\}^{-1}$ $(q = n, p)$ and the quantity ν_q is related to the usual chemical potential μ_q by the equations $\nu_n = \mu_n - g_\omega\omega_0 + g_\rho b_0/2$ and $\nu_p = \mu_p - g_\omega\omega_0 - g_\rho b_0/2$ with $\omega_0 = g_\omega\rho/m_\omega^2$ and $b_0 = g_\rho\rho_3/(2m_\rho^2)$ where $\rho_3 = \rho_p - \rho_n$. The chemical potentials for neutron and proton are determined by their densities: $\rho_q = [2/(2\pi)^3]\int d^3k[n_q(k) - \bar{n}_q(k)]$ $(q = n, p)$. The neutron density ρ_n and proton density ρ_p are also related to the total density ρ and asymmetry parameter α by relations: $\rho_n = (1+\alpha)\rho/2$ and $\rho_p = (1-\alpha)\rho/2$. The effective mass satisfies the following self-consistent equation,

$$1 - m^* = \frac{\alpha C_\sigma^2 C_\omega^2}{M^6} m^{*(\alpha+2\beta)}\rho^2 + \frac{C_\sigma^2}{\pi^2} m^{*(3\beta+1)} \sum_{q=n,p} \int d^3k \frac{1}{E_k}[n_q(k) + \bar{n}_q(k)] \tag{3}$$

Now we discuss the two-phase equilibrium model, which will be used to study the Coulomb instability of hot nuclei. We consider the hot nucleus as a uniformly charged drop of nuclear liquid at a given temperature T and with a sharp edge, in both thermal mechanical and chemical equilibrium with the surrounding vapor. A set of two-phase coexistence equations is, therefore, obtained by requiring equality of temperature T, pressure p, neutron chemical potential μ_n and proton chemical potential μ_p of the liquid and vapor phases:

$$p(T, \rho_L, \alpha_L) + p_{Coul}(\rho_L) + p_{surf}(T, \rho_L) = p(T, \rho_V, \alpha_V) \tag{4}$$

$$\mu_n(T, \rho_L, \alpha_L) = \mu_n(T, \rho_V, \alpha_V) \tag{5}$$

$$\mu_p(T, \rho_L, \alpha_L) + \mu_{Coul}(\rho_L) = \mu_p(T, \rho_V, \alpha_V), \tag{6}$$

where subscripts L and V stand for liquid and vapor, respectively. In the liquid phase, the Coulomb and surface effects have been included. For a uniformly charged sphere, $\mu_{Coul}(\rho) = 6Ze^2/(5R)$ and $p_{Coul}(\rho) = Z^2e^2\rho/(5AR)$ where Z, A and R are, respectively, the charge, mass numbers and radius of the liquid droplet.

The pressure given by the surface tension of the liquid droplet is expressed as $p_{surf}(T, \rho) = -2\gamma(T)/R$. Following Ref[9], the formula for the temperature dependence of the pressure tension $\gamma(T)$ suggested by Goodman *et al.*[13] is used: $\gamma(T) = (1.14\,MeV\,fm^{-2})[1 + 3T/(2T_C)][1 - T/T_C]^{3/2}$, where T_C is the critical temperature for infinite symmetric nuclear matter.

3 RESULTS AND DISCUSSIONS

In the numerical calculations, we will use the coupling constants given in Ref.[11] ($C_\sigma^2 = 219.3, 443.3$; $C_\omega^2 = 100.5, 305.5$ for the ZM2 and ZM3 models, respectively). The coupling constant g_ρ between the ρ meson field and nucleon field in the present model is taken the same as in the usual ZM model,[9] i.e., $C_\rho^2 m^{*2} \equiv (M^{*2}/m_\rho^2)g_\rho^2 = 54.71$, which is determined from $\rho \to 2\pi$ decay.[5]

We will use the EOS given by chemical-density ($\mu-\rho$) isotherms to discuss the liquid-gas phase transition in nuclear matter. The critical temperature T_C can be determined by the condition of the inflection point of $\mu - \rho$ isotherms,

$$(\frac{\partial \mu}{\partial \rho})_T = 0, \qquad (\frac{\partial^2 \mu}{\partial \rho^2})_T = 0 \tag{7}$$

In asymmetric nuclear matter, the proton and neutron are not in chemical equilibrium although they may be in thermal equilibrium[9]. So, their chemical potential are not related to each other. Since the proton and neutron have different chemical potentials, they shall also appear to have different critical temperatures T_C^p and T_C^n, respectively. But we cannot imagine that the kind of nucleons with high critical temperature can stick together after all the other kind of nucleons with lower critical temperature have boiled off. Therefore, We must choose the lower of T_C^p and T_C^n as the correct critical temperature.

For the convenience in the comparison between the results here and those given by the non-relativistic theories, we define a reduced chemical potential $\tilde{\mu}$ as $\tilde{\mu} \equiv \mu - M$. In Fig.1, we present the $\tilde{\mu} - \rho$ isotherms for infinite asymmetric nuclear matter with $\alpha = 0.4$ and at various temperatures, calculated with the ZM2 model. In the case of asymmetric nuclear matter, $\tilde{\mu}_p$ and $\tilde{\mu}_n$ separate. At lower temperatures, both of $\tilde{\mu}_n - \rho$ and $\tilde{\mu}_p - \rho$ isotherms exhibit the form of two-phase coexistence, with an unphysical region for each. When temperature T increases, the unphysical regions get smaller. We can then find a critical temperature T_C^n for neutrons and a critical temperature T_C^p for protons. The result for the asymmetry parameter $\alpha = 0.4$ is $T_C^n = 11.92$MeV and $T_C^p = 21.00$ MeV. As mentioned above , we should choose the lower of the two critical temperatures, T_C^n as the correct critical temperature for asymmetric nuclear matter. For a given asymmetry α, we obtain a critical temperature T_C. Inversely, we obtain a critical asymmetry for a given temperature. The resulted $T_C - \alpha_C$ diagram is shown in Fig.2, calculated with the ZM, ZM2 and ZM3 models. The solid and dashed curves stand for the results with and without the ρ meson degree of freedom. The phase diagram separates the $T - \alpha$ space into two regions. In the exterior region, nuclear matter can exist in gaseous phase only, while in the interior region both liquid and gaseous phases are allowed. For intance, the critical asymmetry at $T = 12$ MeV in the usual ZM model without the ρ meson degree of freedom is $\alpha = 0.84$, above which only the gaseous phase can exist in nuclear matter.

It is seen that the critical temperature T_C decreases monotonously as the asymmetry parameter α increases in all the three ZM models. The critical temperature in different ZM models decreases in the sequence: ZM, ZM2 and ZM3 in any asymmetry case. Now let us discuss the effect of including the ρ meson degree of freedom. One can find that the inclusion of the ρ meson degree of freedom gives an additional asymmetry effect on the chemical potentials μ_q and pressure p. As a result, the chemical potential of neutron shifts up and the chemical potential of proton shifts down further. The increase in the gap between μ_n and μ_p shall decrease the critical temperature in asymmetric nuclear

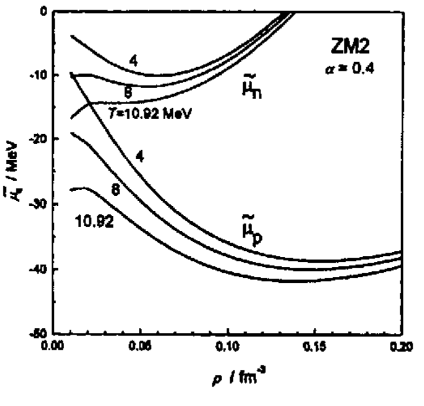

Fig.1 $\tilde{\mu}_n - \rho$ and $\tilde{\mu}_p - \rho$ isotherms of infinite asymmetric nuclear matter with the asymmetry parameter $\alpha = 0.4$ and at various temperatures, calculated with the ZM2 model

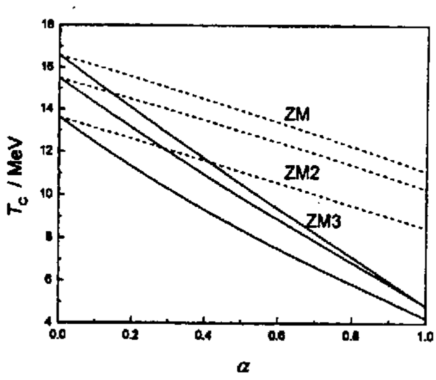

Fig.2 Phase diagram of the critical temperature T_C plotted against the critical asymmetry parameter α_C for infinite nuclear matter, calculated with the ZM, ZM2 and ZM3 models with (solid curves) and without(dashed curves) the ρ meson

matter. This result is explicitly shown in Fig.2. The curves with the ρ meson drop more quickly than those without the ρ meson.

Next, we discuss the Coulomb instability of hot nuclei by calculating the limiting temperature T_{lim}, above which the set of coexistence equations (4~6) has no solution. We show in Fig.3 by solid (dashed) curves the mass number dependence of the limiting temperature T_{lim} in the ZM, ZM2 and ZM3 models with (without) the ρ meson for the nuclei along the β-stability line:

$$Z = 0.5A - 0.3 \times 10^{-2} A^{5/3} \qquad (8)$$

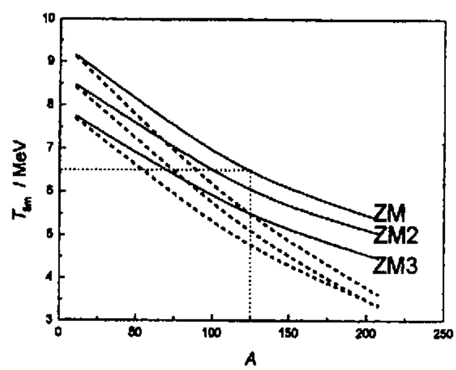

Fig.3 Mass number dependence of limiting temperature T_{lim} calculated by using the ZM, ZM2 and ZM3 models with(solid curves) and without (dashed curves) the ρ meson

It is seen from Fig.3 that the six curves have a similar trend: the limiting temperature decreases monotonously as the mass number A increases, but the rate of the decrease is smaller for larger A. It is also seen that the limiting temperatures have the same sequence as the critical temperature for different ZM models. This result indicates that the hot nuclei described by the ZM model is more stable than that described by the ZM2 or ZM3 models. This conclusion is consistent with the calculated critical temperatures of asymmetric nuclear matter (see Fig.2). Considering the different values of compression modulus in different ZM models: 224.7 MeV for ZM; 198.3 MeV for ZM2 and 155.7 MeV for ZM3, we find a rule:the softer the matter, the lower the T_C and T_{lim}. We have also noticed that the limiting temperature T_{lim} calculated with the ρ meson is always higher than the corresponding one without the ρ meson and exhibits explicitly an additional asymmetry effect. As a result, the heavier hot nucleus becomes more stable when the ρ meson degree of freedom is included in the Lagrangian of the system. We have also examined the equilibrium values of the asymmetry parameter α_V for vapor phase and found that they are always negative when without the ρ mesons in the system. The result is not reasonable in physics and is very different from that in the non-relativistic theories,[14] where the asymmetry parameter of vapor is always positive. When the ρ meson degree of freedom is included, however, the situation changes, i.e., the asymmetry α_V in the vapor phase becomes positive. It indicates that it is necessary to include the ρ meson degree of freedom in the description of asymmetric nuclear system. We have also shown in Fig.3 an experimental point ($T_{lim} = 6.5\,\mathrm{MeV}$ for $A \simeq 125$) given by Natowitz *et al*,[15] who summarized their early experimental results.[16] We found that this experimental point is very close to the result predicted by the usual ZM model. The results given by the ZM2 and ZM3 models is 1 or 2 MeV lower than the experimental data.

References

1 Siemens P J. Nature(London), 1983, **305**:410
2 Elliot J B *et al.* Phys Rev, 1994, **C49**:3185
3 ALADIN/LAND Collaboration, Pochodzalla J *et al.* Phys Rev Lett, 1995, **75**:1040
4 Levit S, Bonche P. Nucl Phys, 1985, **A437**:426
5 Serot B D, Walecka J D. In: Advances in Nuclear Physics, Negele J W, Vogt E ed. Vol. 16, (New York: Plenum, 1986)
6 Boguta J, Bodmer A R. Nucl Phys, 1977, **A 292**:413
7 Zimanyi J, Moszkowski S A. Phys Rev, 1990, **C42**:1416
8 Saito K, Thomas A W. Phys Lett, 1994, **B327**:9
9 Song H Q, Qian Z X, Su R K. Phys Rev, 1994, **C49**:2924
10 Delfino A, Coelho C T, Malheiro M. Phys Rev, 1995, **C51**:2188
11 Malheiro M, delfino A, Coelho C T. Phys Rev, 1998, **C58**:426
12 Chiapparini M, Delfino A, Malheiro M *et al.* Z Phys, 1997, **A357**:47
13 Goodman A L, Kapusta J I, Mekjian A Z. Phys Rev, 1984, **C30**:851
14 Song H Q, Su R K. Phys Rev, 1991, **C44**:2505
15 Natowitz J B *et al.* Phys Rev, 1995, **C52**:R2322
16 Wada R *et al.* Phys Rev, 1989, **C39**:497