Simulation study of transverse optical klystron radiation*

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Abstract The radiation from a transverse optical klystron (TOK) is calculated by far field approximation equation and numerical integration, in which the effects of electron-beam emittance and energy spread are considered. Accurate electron-beam profiles have been experimentally determined and modeled by the Monte Carlo method. The calculated spectra illustrate the emittance of Hefei storage ring imposes on the spontaneous radiation of TOK.

Keywords Spontaneous emission, Monte Carlo simulation, Emittance, Energy spread

1 Introduction

The 2 m long TOK[1] (shown as Fig.1) is made for generating coherent harmonic of short wave length to meet the requirement of experimental study on the electron storage ring in Hefei. Before the TOK operates in storage ring, the spontaneous radiation spectra are numerically simulated.

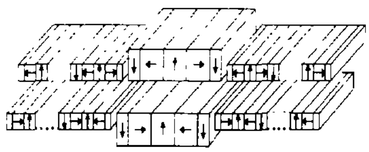

Fig.1 The structure of the TOK

We developed software to calculate the spectra of insertion device having an arbitrary magnetic-field distribution, including the affects of finite electron-beam emittance and energy spread. Realistic modeling of electron beam is an essential requirement. Thus, electron-beam profiles should be determined experimentally and used in the calculations. We modeled the observed electron-beam profiles using a Monte Carlo method. In this paper we report briefly the modeling of electron beam, and present the calculation spectra influenced by the properties of electron beam.

2 Computation of the spontaneous radiation

Let us now consider independent N electrons in arbitrary motion and let $\vec{r_j}\ (t)$ and $\vec{\beta_j}(t)$ be the position and the velocity of the electron j at time t. We assume that the electron radiations are independent of each other. In far field approximation, the general form of the power radiated per solid angle $\Delta\Omega$, in the direction $\vec{n_j}$, in the frequency range $\Delta\omega$, and in terms of an integral over the trajectory of an electron is[2]

$$\frac{\mathrm{d}^2 I}{\mathrm{d}\omega \mathrm{d}\Omega} = \frac{e^2\omega^2}{16\pi^3\varepsilon_0 c}\left|\int_{-\infty}^{+\infty} \vec{A}_j \exp\left[i\omega\left(t - \frac{\vec{n}_j\ (t)\cdot \vec{r_j}\ (t)}{c}\right)\right]\mathrm{d}t\right|^2 \tag{1}$$

with $\vec{A_j}=\vec{n}_j\ (t)\times\ \vec{n}_j\ (t)\times\ \vec{\beta_j}(t)$

We may assume that $\vec{n}_j$ is a constant independent of t. For a distance detector at spherical polar angles θ and φ with respect to the z axis, i.e. ,

$$\vec{n}= \sin\theta\sin\varphi\widehat{x} + \sin\theta\cos\varphi\widehat{y} + \cos\theta\widehat{z} \tag{2}$$

and then Eq.(1) becomes the following approximation equation:

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$$\frac{\mathrm{d}^2 I}{\mathrm{d}\omega \mathrm{d}\Omega} = \frac{e^2\omega^2}{16\pi^3\varepsilon_0 c}\left|\int_{-\infty}^{+\infty}(A_x\widehat{x} + A_y\widehat{y} + A_z\widehat{z})\exp\left[i\omega\left(t - \frac{\vec{n}_j\cdot\vec{r}_j}{c}\right)\right]\mathrm{d}t\right|^2 \tag{3}$$

with

$$\begin{aligned}
A_x &= \beta_x \sin^2\theta \sin^2\varphi + \beta_y \sin^2\theta\cos\varphi\sin\varphi + \beta_z \sin\theta\sin\varphi\cos\theta - \beta_x \\
A_y &= \beta_x \sin^2\theta\cos\varphi\sin\varphi + \beta_y \sin^2\theta\cos^2\varphi + \beta_z\sin\theta\cos\varphi\cos\theta - \beta_y \\
A_z &= \beta_x\sin\theta\sin\varphi\cos\theta + \beta_y\sin\theta\cos\varphi\cos\theta + \beta_z\cos^2\theta - \beta_z
\end{aligned} \tag{4}$$

The position integral of Eq.(3) needs the detailed trajectory. When the electron's starting position and divergence relative to the ideal orbit are known, the electron trajectory can be calculated by solving the associated differential equation using the measured magnetic field of the TOK[3].

3 Realistic modeling of the electron beam

Taking the beam emittance and energy spread effects into consideration, we use the Monte Carlo method in the numerical calculation. The Monte Carlo method realistically models the actual physical system. Let us consider the distribution of electron displacements and angular divergences from the ideal electron orbit. The total radial displacement x (and similarly, the divergence x') from the ideal electron orbit is the sum of two parts: the betatron displacement x_β respect to the closed orbit and the displacement x_e of the off-energy closed orbit.[4] Consequently, one writes,

$$x = x_\beta + x_e, \qquad x' = x'_\beta + x'_e \tag{5}$$

It is assumed that the betatron displacement and betatron slope have a standard binomial distribution in both the horizontal (x) and vertical (y) directions. The probability distribution in one plane (either x or y) is [5,6]

$$P(x_\beta, x'_\beta) = \frac{1}{2\pi\sigma_{\beta x}\sigma'_{\beta x}\sqrt{1-\rho^2}}\exp\left[-\frac{1}{2(1-\rho^2)}\left(\frac{x_\beta^2}{\sigma_{\beta x}^2} - 2\rho\frac{x_\beta x'_\beta}{\sigma_{\beta x}\sigma'_{\beta x}} + \frac{x'^2_\beta}{\sigma'^2_{\beta x}}\right)\right] \tag{6}$$

where:

$$\sigma_{\beta x} = \sqrt{\beta_x\varepsilon_x}, \qquad \sigma'_{\beta x} = \sqrt{\gamma_x\varepsilon_x}, \qquad \rho = 1 - \left(\frac{\varepsilon_x}{\sigma_{\beta x}\sigma'_{\beta x}}\right)^2$$

ε_x is the emittance, β_x the amplitude function and γ_x is the Twiss parameter. Note that $\left(x, x'\right)$ in these equations can be replaced equally well by (y, y') .

When the energy of an electron changes, there result in electron displacement x_e, and angular divergence x'_e given by[4]

$$x_e = \eta_x\frac{\varepsilon}{E}, \qquad x'_e = \eta'_x\frac{\varepsilon}{E}, \tag{7}$$

where η is the dispersion function. The probable density $W(\varepsilon)$, where ε is an energy deviation from the most probable value of an energy E, is a distribution according to

$$W(\varepsilon) = \frac{1}{\sqrt{2\pi}\sigma_\varepsilon}\exp\left(-\frac{\varepsilon^2}{2\sigma_\varepsilon^2}\right) \tag{8}$$

The parameter σ_ε is the root-mean square, often called the standard deviation, of the energy distribution.

It is possible to generate an electron-beam model from the probability densities given by Eqs.(6) and (8) with Twiss parameters, the dispersion functions, the beam emittance, and the energy spread. Although not measured directly, those parameters can be determined from theoretical calculation and experimental observation. We adopted calculated values of the Twiss parameters and dispersion functions from the computer code.

At the entrance of the TOK, electron beam phase space can be simulated by the standard Monte Carlo method:

$$x = x_0 + \sigma_x\sqrt{-2\ln u}\left(\sqrt{1-\rho^2}\cos 2\pi v + \rho\sin 2\pi v\right) \tag{9a}$$

$$x' = x_0' + \sigma_x'\sqrt{-2\ln u}\sin 2\pi v \tag{9b}$$

where u, v are random variables and uniformly distributed between(0, 1).

The beam size measurements were carried out on Hefei storage ring (at 200 MeV) at one of the bending magnets. The synchrotron radiation from the bending magnet was focused with a lens and the transverse beam profile was imaged on the detection surface of a CCD camera[7]. Fig.2 shows the CCD image of the synchrotron radiation from the bending magnet. The transverse beam sizes in the horizontal and vertical directions (σ_x, σ_y) were then calculated from the CCD image. Combining the calculated values of the Twiss parameter, we can calculate the emittances which are $\varepsilon_x = 1.27\times10^{-7}$ mrad and $\varepsilon_y = 8.45\times10^{-8}$ mrad. Then the transverse beam sizes at entrance of the TOK can be calculated from the emittances and the calculated values of the Twiss parameter. Therefore the source points at the bending magnet and TOK are characterized by the parameters given in Table 1. And the energy spread σ_ε/E is measured to be 0.05% approximately.[8]

x_c and x_c' can be neglected because σ_ε/E is much small. Fig.3 shows the electron-beam model at the entrance of the TOK, and Fig.4 shows the spread of electron-beam simulated by the Monte Carlo method.

Table 1 The characteristics of the bending magnet and TOK

	σ_x/mm	σ_y/mm	β_x/m·rad^{-1}	β_y/m·rad^{-1}
Bend magnet	0.677	1.028	2.4	20.86
TOK	1.63	0.59	12.5	4.09

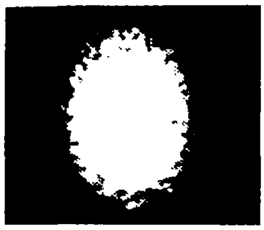

Fig.2 CCD image of the synchrotron radiation from the bending magnet

Fig.3 Monte Carlo simulation of the electron beam in (a) horizontal and (b) vertical phase space at the entrance to the TOK at ring energy of 200 MeV

4 Results and discussion

Fig.5 shows the simulation result of the first harmonic spontaneous emission for an optical klystron, with electron beam emittances. The calculated spectra show the asymmetry of the modulation rate, which is typical of this kind of dispersion and in which the wavelength emitted by an electron is proportional to θ^2; consequently, the side corresponding to higher wavelength is jumbled. According to theoretical result, the modulation rate can be calculated from the following formula:[2]

$$f_i = \exp\left[-\frac{k_i}{2}\left(1+2\frac{\theta_i^2}{\sigma_{\theta i}^2}\right)\right] \qquad (10)$$

where i stands for x or y direction; $k_i = \pi(d+L)\sqrt{2}\sigma_{\theta i}^2/\lambda$. This result can be used to measure the angular spread of the beam by simply measuring the modulation rate on the spontaneous emission off-axis from the electron beam.

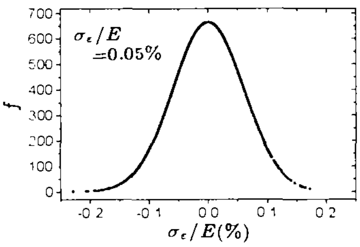

Fig.4 Monte Carlo simulation of the electron beam in energy spread

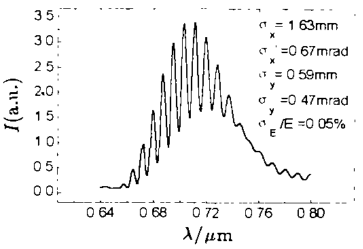

Fig.5 Spontaneous emission for the TOK including electron beam emittance and energy spread

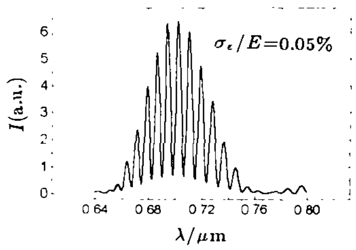

Fig.6 Spontaneous emission for the TOK including only electron beam energy spread

On the other hand, the energy spread induces an inhomogeneous decrease in the modulation rate for the spontaneous emission in the TOK. Fig.6 illustrates the influence of electron beam energy spread on the spontaneous emission spectra. The modulation rate imposed by energy spread can be calculated from the following formula:[2]

$$f = \exp\left[-8\pi^2 (N+N_d)^2 \left(\frac{\sigma_\varepsilon}{E}\right)^2\right] \qquad (11)$$

Therefore, the electron beam energy spread can be determined by simply measuring the modulation rate of spontaneous emission spectra.

The simulation results include the influence of the transverse distribution of magnetic field and field errors. In realistic measurement, the spectra include these influences also, and these influences should be subtracted. Thus the measurement of spontaneous emission spectra provides a way to diagnose electron beam.

The simulation results show that the properties of electron beam in Hefei storage ring impose sharply on the spontaneous spectra of TOK , and limit the effective gain of TOK.

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