Improved quasi-static nodal Green's function method*

Jing Xing-Qing[1](经荥清), Li Jun-Li[1](李君利) and Hu Da-Pu[2](胡大璞)
([1]*Institute of Nuclear Energy and Technology,* [2]*Department of Physics Engineering, Tsinghua University, Beijing 102201*)

Abstract To solve the multi-dimensional transient neutron diffusion equations, improved quasi-static Green's function method (IQS/NGFM) is adopted to deal with the temporal problem, which will increase the time step as long as possible so as to decrease the number of times of spatial calculation. The time step of IQS/NGFM can be increased to 5 $\sim$ 10 times longer than that of full implicit differential method. In spatial calculation, the theory of NGFM is used to get the distribution of shape function, with coarse meshes which can be nearly 20 times larger than that of traditional finite differential method. So the IQS/NGFM is considered as an efficient kinetic method.

Keywords Reactor kinetics, Stiffness, Transient problems

1 Introduction

The research on reactor kinetics is essential to the efficient design and reliable safety analysis of modern nuclear reactor systems. With the development of nuclear industry, the accident analysis for nuclear reactor is required to give more detailed information about the reactor core, so the research on multi-dimensional transient problems has become the main developing trend in reactor kinetics.

In the past decades, several different kinetic methods have been developed, such as direct differential method, synthesis techniques, point kinetics model and quasi-static method. But most of these methods can not satisfy requirements for the solving of multi-dimensional transient problems. For example, the direct differential method has very simple forms of formulae, but its time steps are confined seriously by the stiffness property of transient neutron diffusion equations; although point kinetics model has been applied over a number of past years, it can not be successfully used unless the distortion of neutron flux is very small and a slight deviation from critivity occurs in the reactor core; moreover, for the synthesis techniques, it is of very limited use in transient analysis for its too much dependence on the choice of trail functions. As an attempting to find an efficient kinetic method, a new nodal method named improved quasi-static nodal Green's function method (IQS/NGFM) is established and evaluated in this paper. It mainly overcome two difficulties in the solution of transient problems: One is the stiffness property of the transient neutron diffusion equations, the other is the large number of meshes in spatial calculations wasting the most of computing time. In this new method of IQS/NGFM, improved quasi-static method is adopted as a basic method: The neutron flux is factorized into shape function and amplitude function, and the main change of neutron flux is concentrated on the amplitude function so that the shape function can be determined by large time steps. Meanwhile, in the spatial calculations for shape function, the theory of NGFM is introduced to enlarge the mesh-intervals in space, so the number of mesh points can be greatly decreased. Therefore, the NGFM is of a computing speed which is nearly 1000 times faster than that of infinite differential method. Because of the unification of IQS and NGFM, the IQS/NGFM method possesses higher efficiency to solve the space-time kinetics equations.

A normal derivation of the IQS/NGFM method for multi-group transient neutron diffusion equations is carried out in Sec.2. To evaluate the potential of the method for kinetic problems, the results of two benchmark problems are presented in Sec.3. Finally, a brief discussion and conclusion is made in Sec.4.

*The Project Supported by National High Technology "863" Plan
Manuscript received date: 1995–10–20

2 Formulation of the method

The essence of improved quasi-static nodal Green's function method is the factorization in which the neutron flux is changed into shape function and amplitude function. IQS/NGFM mainly consists of the following three steps.

2.1 Factorization

The multi-dimensional transient neutron diffusion equations can be written in standard multi-group form:

$$\frac{1}{V_g}\frac{\partial\phi_g(r,t)}{\partial t} = \nabla\cdot D_g(r)\nabla\phi_g(r,t) - \Sigma_g^R(r)\phi_g(r,t) + \sum_{g'=1}^{g-1}\Sigma_{gg'}^s(r)\phi_{g'}(r,t)$$

$$+(1-\beta)\chi_g\sum_{g'=1}^{G}\nu\Sigma_{fg'}(r)\phi_{g'}(r,t) + \sum_{i=1}^{Id}\lambda_i\chi_{ig}C_i(r,t) \tag{1a}$$

$$\frac{\partial C_i(r,t)}{\partial t} = \beta_i\sum_{g'=1}^{G}\nu\Sigma_{fg'}(r)\phi_{g'}(r,t) - \lambda_i C_i(r,t); \quad g=1,2,\ldots,G; I=1,2\ldots,I_{\mathrm{d}} \tag{1b}$$

For the equations above that the parameters are expressed with standard symbols, a factorization is introduced[1]. That is

$$\begin{cases}\phi_g(r,t) = n(t)\psi_g(r,t)\\ C_i(r,t) = C_i^*(t)\psi_i(r,t)\end{cases} \tag{2}$$

$$i=1,2,\ldots,I_{\mathrm{d}}$$

where the $\psi_{\mathrm{g}}(r,t)$, $\psi_i(r,t)$ are the time- and space-dependent shape functions, respectively for the neutron flux and the concentration of delayed neutron precursors, and the $n(t), C_i^*(t)$ represent their time-dependent amplitude functions. To deal with the influence of delayed neutron precursors in the case of transient exactly, here, the concentration of delayed neutrons is factorized as well. By substituting Eq.(2) into Eq.(1), the shape function equations can be obtained. That is

$$\frac{1}{V_g}\frac{\partial\psi_g(r,t)}{\partial t} = \nabla\cdot D_g(r)\nabla\psi_g(r,t) - [\Sigma_g^R(r) + \frac{1}{n(t)V_g}\frac{\mathrm{d}n(t)}{\mathrm{dt}}]\psi_g(r,t) + \sum_{g'=1}^{g-1}\Sigma_{gg'}^s(r)\psi_{g'}(r,t)$$

$$+(1-\beta)\chi_g\sum_{g'=1}^{G}\nu\Sigma_{fg'}(r)\psi_{g'}(r,t) + \sum_{i=1}^{Id}\lambda_i\chi_{ig}\frac{C_i^*(t)}{n(t)}\psi_i(r,t) \tag{3a}$$

$$\frac{\partial\psi_i(r,t)}{\partial t} = \beta_i\sum_{g'=1}^{G}\nu\Sigma_{fg'}(r)\psi_{g'}(r,t)\frac{n(t)}{C_i^*(t)} - (\lambda_i + \frac{1}{C_i^*(t)}\frac{\mathrm{d}C_i^*(t)}{\mathrm{dt}})\psi_i(r,t) \tag{3b}$$

Using the adjoint neutron flux of steady-state ϕ_g^* and $\phi_g^*\ \chi_{ig}$ as weight functions, Eq.(3) can be integrated over all space with a sum to energy groups made previously. Then, shape functions can be constrained by introducing the normalization:(Here, the space ordinates are omitted for convenience)

$$\Sigma_{g=1}^G < \phi_g^*\frac{1}{V_g}\psi_g >= 1 \tag{4a}$$

$$\Sigma_{g=1}^G < \phi_g^*\chi_{ig}\psi_i >= 1 \tag{4b}$$

$$i=1,2,\ldots,I_{\mathrm{d}}$$

The amplitude function equations can be got as follows:

$$\frac{\mathrm{d}n(t)}{\mathrm{d}t} = \frac{\rho - \overline{\beta}}{\Lambda} n(t) + \sum_{i=1}^{Id} \lambda_i C_i^*(t) \qquad (5a)$$

$$\frac{\mathrm{d}C_i^*(t)}{\mathrm{d}t} = \frac{\overline{\beta}_i}{\Lambda} n(t) - \lambda_i C_i^*(t) \qquad (5b)$$

$$g = 1, 2, \ldots, G;\ i = 1, 2, \ldots, I_{\mathrm{d}}$$

where the parameters ρ, Λ , etc., are all functions of ψ_g.Therefore, the transient equations are changed into two groups of equations: the shape function equations and the amplitude function equations, which are entirely equivalent to the former equations.

2.2 Amplitude function equations

If the time- and space-dependent shape function $\psi_g(r,t)$ is known, the parameters of amplitude function equations such as ρ and Λ will be determined. Then, the shape function equations are equal to the "point kinetic" equation possessing a special property of stiffness in temporal domain. The "point kinetics" equation for a thermal reactor is moderately stiff, and for a fast-breeder reactor appears a strong stiffness character. Due to the stiffness problem, the time steps for the solution of "point kinetics" equations must be short enough to satisfy the demands required by the numerous stability in numerical calculations. There are several different kinds of approaches developed for solution of "point kinetics" equations. In this paper, PpqM[2] is adopted, which approximated the unknown with piecewise polynomials of order $\mathcal{O}(p+q+1)$. Compared with other approaches which have been used for "point kinetics" equations such as Gear method and three-order Hermite polynomials method[3] , PpqM method indicates a reduction in computerizing time and high precision of computation. In addition, this method is A -stable at least in numerous calculations, so it is still effective even for the fast-breeder reactor models.

2.3 Shape function equations

In this paper, for the solution of the multi-dimensional shape function equations, the theory of the nodal Green's function method[4] is introduced. This method is based on the linear form of the nodal balance equation written in terms of the average partial currents across the surfaces of the node. By spatially integrating the multi-dimensional shape function equations over directions transverse to each coordinate direction, a coupled set of one-dimensional integral equations of shape function can be obtained. These equations will analytically provide the necessary additional relationships between the interface partial currents and the flux within the node, when solved in conjunction with the nodal balance equations. As the result of the introduction of Green's function, NGFM is of the capability to yield very accurate results in significantly smaller computing time than those required by standard finite differential methods, with coarse meshes in space.

3 Numerical results

To gain some insight into the computing efficiency, the IQS/NGFM method has been applied to several two-and three-dimensional light water reactor benchmark problems. In this section, we present some results of two benchmark models by IQS/NGFM.

3.1 Two-dimensional TWIGL model[5]

The TWIGL benchmark problem is a simplified two-dimensional model consisting of two different fuel-zones (the model explanation refers to Ref.[5]). During the course of 0.0$\sim$ 0.2 s , the power in core increases gradually with time as the result of an reactivity introduced into the reactor core. For the geometry symmetry of the model, an calculation for quarter core is enough, with 8-cm meshes in space. The reference solution of this model is obtained by the full implicit differential method with very short time steps.

Table 1 demonstrates that the time intervals for IQS/NGFM can be roughly 20 times larger than that for NGFM, without any loss in calculation precision. The maximum relative error of relative power density by NGFM is 8.2×10^{-4}, however that of IQS/NGFM is only 1.0×10^{-4}. In the view of relative flux level in the central node of quarter core, which indicates the accuracy gained in spatial calculation, the IQS/NGFM is more accurate than NGFM as well, even with much larger time steps. However, the execution time of IQS/NGFM is only one-sixteenth that of NGFM.

3.2 Three-dimensional LMW Model[5]

This two-group problem represents an op-

erational transient in a model LWR with 20cm square fuel assemblies(the explanation of the model refers to Ref.[5]. The transient is initiated by withdrawing a bank of partially inserted control rods at a rate of 3 cm/s over $0 \leq t \leq 26.7$s. The second bank of control rods is inserted at the same rate over $7.5 \leq t \leq 47.5$s. As a result of the movements of these two groups of control rods, the reactor power density has an increase at first, then descends gradually because of the insertion of the second bank of control rods, therefore, has a peak at about 20.0s. (see Fig.1) The reference solution of LMW model is obtained by NGFM with very short time intervals, and the assembly-size(20-cm) coarse meshes are used.

Table 1 Comparison between the results of TWIGL by IQS/NGFM and NGFM when comparable accuracy is obtained

	NGFM Δt=0.005s				IQS/NGFM $\Delta t_1 = 0.1$s $\Delta t_2 = 0.1$s $\Delta t_3 = 0.3$s			
t/s	Relative power density	Relative errors	Relative flux level in central node	Relative errors	Relative power density	Relative errors	Relative flux level in central node	Relative errors
0.0	1.0000	0.0	1.0000	0.0	1.0000	0.0	1.0000	0.0
0.1	1.3086	3.1×10^{-4}	1.3415	3.0×10^{-4}	1.3083	7.6×10^{-5}	1.3412	7.5×10^{-5}
0.2	1.9607	8.2×10^{-4}	2.0615	8.3×10^{-4}	1.9593	1.0×10^{-4}	2.0600	9.7×10^{-5}
0.3	2.0737	-4.8×10^{-5}	2.1802	0.0	–	–	–	–
0.4	2.0911	0.0	2.1984	0.0	–	–	–	–
0.5	2.1086	0.0	2.2168	0.0	2.1086	0.0	2.2168	0.0
Execution time(Acer486DX2/66)/s			244				15	

As indicated in Table 2, the IQS/NGFM results show the maximum relative error of -5.07×10^{-4} in power density, roughly ten times lower than that of NGFM, when the time steps for these two methods are all equal to 5.0s.

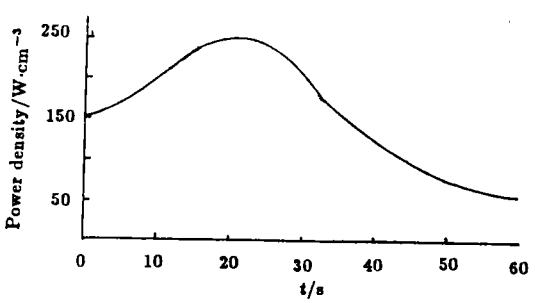

Fig.1 Power density curve for the three-dimensional LMW model

Table 3 demonstrates that for comparable relative errors, the time steps that can be used by the IQS/NGFM are roughly five times larger than those required by the NGFM. On the other hand, execution time for the IQS/NGFM is 148s, that for NGFM is 317s. This means that the IQS/NGFM seems to be nearly twice as fast as the NGFM in speed for the solution of this three-dimensional LMW model. Therefore, the IQS/NGFM possesses a higher calculating efficiency, compared with the NGFM.

Why the execution time saving for LMW model by the IQS/NGFM is less than that achieved for TWIGL model mainly includes two reasons: firstly, the IQS/NGFM requires one more steady-state calculation than the NGFM, while there is much more time needed for three-dimensional spatial calculations to get enough computational accuracy. So some of the execution time saved by the way of enlarging time steps in IQS/NGFM are compensated. Secondly, the IQS/NGFM needs more iterations in space to solve the shape function equations, when the time steps are enlarged.

Finally, it should be emphasized that the comparison of calculating efficiency above only expressed the effects obtained by the introduction of IQS, not including the attractive effects of the improvement from fine-mesh calculation to NGFM which has a much higher speed

than the direct differential method. Taking this into account, the advantages of IQS/NGFM in calculation will be more remarkable, compared with other space-time kinetic methods which have been used in the past time.

Table 2 Comparison between the results of LMW by NGFM and IQS/NGFM when Δt is equal to 5.0s

	NGFM		IQS/NGFM	
t/s	APD/W.cm^{-3}	RE	APD/W.cm^{-3}	RE
0.0	150.00	0.0	150.00	0.0
10.0	200.74	7.73×10^{-3}	199.10	-5.07×10^{-4}
20.0	252.04	-2.93×10^{-3}	252.87	3.48×10^{-4}
30.0	198.79	-1.57×10^{-3}	201.91	-2.48×10^{-4}
40.0	118.94	-6.43×10^{-3}	119.68	-2.17×10^{-4}
50.0	74.83	3.76×10^{-3}	74.53	-3.22×10^{-4}
60.0	57.36	1.60×10^{-3}	57.27	0.0

Table 3 Comparison between the results of LMW for NGFM and IQS/NGFM when the comparable accuracy is obtained

	NGFM Δt = 1.0s		IQS/NGFMΔt = 5.0s	
t/s	APD/W.cm^{-3}	RE	APD/W.cm^{-3}	RE
0.0	150.00	0.0	150.00	0.0
10.0	199.25	2.5×10^{-4}	199.10	-5.07×10^{-4}
20.0	252.69	-3.6×10^{-4}	252.87	3.48×10^{-4}
30.0	201.86	-5.0×10^{-4}	201.91	-2.48×10^{-4}
40.0	119.71	0.0	119.68	-2.17×10^{-4}
50.0	74.59	5.4×10^{-4}	74.53	-3.22×10^{-4}
60.0	57.29	3.5×10^{-4}	57.27	0.0
Execution time (Acer DX2/66)	for steady-state: 15s for transient: 30s total: 317s		for steady-state: 32s for transient 116s total:148s	

Notes: APD=average power density, RE=Relative error

4 Conclusion

A nodal method with improved quasi-static method for the solution of multi-dimensional kinetic problems has been presented and numerically tested in this paper. In this method, the time steps can be enlarged by the factorization of neutron flux. Meanwhile, in spatial calculation, the theory of NGFM is introduced which is of the capability to increase the mesh intervals in space. The test results of benchmark models(i.e. two-dimensional TWIGL and three-dimensional LMW) demonstrate that the IQS/NGFM possesses a high efficiency in the solution of space-time kinetic problems.

Acknowledgement

The authors would like to express their appreciation to Prof. Zhang Yuman, Prof. Shan Wenzhi and Prof. Hu Yongming for their kind help and useful discussions

References

1 Hu Dapu. Nucl Sci Eng (Chinese edition), 1994; 15(1):45-54
2 Hernart J P. Nucl Sci Eng, 1977; 64:875-901
3 Yeh Kueng. Nucl Sci Eng, 1978; 66:235-242
4 Lawrence R D. Nodal Green's function method for the multi-dimensional neutron diffusion calculations. Ph.D thesis, University of Illinois, Urbara, USA, 1979
5 Smith K S. An analytic nodal method for solving the two-group, multidimensional, static and transient neutron diffusion equations. MS thesis, Massachusetts Institute of Technology,1979

Prof. Jing Xing-Qing, graduated from Department of Mathematics and Mechanics, Tsinghua University in 1963, has been engaged in reactor physics design and calculation for many years, including transport equations, diffusion equations and space-time kinetics, now works in the Institute of Nuclear Energy and Technology in Tsinghua University, in charge of the physical design of HTR10.