Calculation of mean dose deposited in extended volume around an ion path*

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Abstract Using the relation of radial dose distribution which is inverse proportion to square of radial distance, and considering angular distribution of secondary electrons, an analytical formula of mean dose deposited in extended volume around an ion is given and the inactivation cross sections of heavy ions are calculated. The results are in reasonable agreement with experimental data. Compared to the numerical integral methods, the method using analytical formulae is straightforward and simple.
Keywords Track structure theory. Radial dose distribution, Inactivation cross section

1 Introduction

The distribution of mean dose in an extended volume taken approximately as the size of a sensitive element in a biological cell around a charged particle is very important to the quantitative interpretation of biological heavy-ion effects. By assuming that the response of small subvolumes (volume of a sensitive element) around an ion path is as if these subvolumes were uniformly irradiated with gamma-ray at the same dose, the effects produced in charged particles irradiation may be related with those observed in gamma-ray irradiation.[1,2] Thus, the response of a medium to gamma-ray can be coupled to the spatial distribution of secondary electron dose to yield the spatial distribution of response about the path of single charged particle. The radial integral of the inactivation probability is the inactivation cross section. It is called track structure theory.[3] In general, the sensitive element is represented by a short cylinder whose axis is parallel to the ion path, the mean dose in the cylinder is calculated by numerical integration method. When calculating the inactivation cross sections for the heavy ion bombardments of V-97 Chinese hamster cells using the track theory, the thindown effect that cross section decreases with an increase in stopping power accompanying a decrease in energy of a bombarding ion is good description. The calculated cross sections are in agreement with experimental results by multiplying a factor 20 and adjustment of the range of secondary electrons.[4] Considering angular distribution of secondary electrons and a logarithmic polynomial representation of electron range-energy relation, the cross sections of V-97 cells are calculated. The results found that the adjustment in range of secondary electron is not needed.[5] But the numerical integration is rather complicatedly. Considering angular distribution of secondary electrons, we give an analytical formula of mean dose deposited in extended volume around an ion and the inactivation cross sections of heavy ions are calculated using the formula. The results are in reasonable agreement with experimental data. Compared to the numerical integral methods, using analytical formulae is straightforward and simple.

2 Theoretical description of the calculation method

When the extended volume is represented by a short cylinder of radius a_0 whose axis is parallel to the ion path, the mean dose deposited in the extended volume

$$\overline{D}(t) = \frac{1}{\pi a_0^2}\int_0^{a_0}\int_0^{2\pi} D(R) r \mathrm{d}\theta \mathrm{d}r \qquad (1)$$

where t is the radial distance from the ion path to the axis of the cylinder, $R = (t^2 + r^2 + 2rt\cos\theta)^{1/2}$, and $D(R)$ is point distribution of radial dose around an ion path.

Based on different assumptions, there are a number of models proposed to deal with the

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radial distribution of dose about the ion path.[1,6~9] They all agree that $D(R)$ varies with R as $1/R^2$ approximately when R is not very near T_{max}, the maximum radial distance of secondary electrons from an ion path, and there is a steeper decline as R is very close to T_{max}. For practical purposes it may be assumed that there is sudden cut-off at T_{max}. The radial distribution of dose around an ion path can then be expressed as[9]

$$D(R) = C\frac{Z_{eff}^2}{\beta^2}R^{-2} \quad (2)$$

for $R \leq T_{max}$, and $D(R) = 0$ for $R > T_{max}$, where C is a coefficient which only depends on the absorbing medium and $C = 1.25 \times 10^{-8}$ erg/cm for water, β is ion velocity relative to that of light in vacuum and Z_{eff} is the effective ion charge number. Considering the ejecting angle of secondary electrons in water, the maximum distance of secondary electrons from an ion path is[9]

$$T_{max} = 6.16^{-2}(E/M_i)^{1.7} \quad (3)$$

where T_{max} is measured in μm, E is the energy of an ion in MeV and M_i is the mass of an ion in atomic mass units.

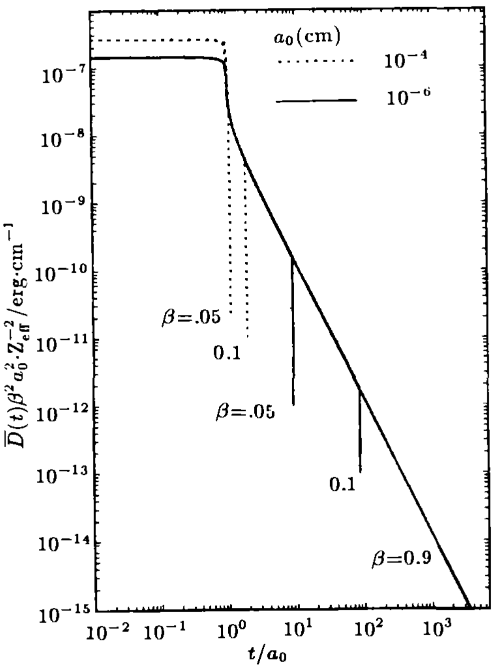

Fig.1 The mean dose $\overline{D}(t)$ deposited by secondary electrons in a cylinder sensitive element of radius a_0, whose center is at radial distance t from the path of an ion with effective charge number Z_{eff}, moving at relative speed β

Using the above equations, the mean dose deposited in an extended volume is calculated. To avoid the divergence during calculaton, we replace $1/R^2$ with $1/(R^2 + \epsilon^2)$ in Eq.(2), and ϵ is set to $3.5\times10^{-5}\mu$m that is about the range of 1 eV electrons in water. The result is

$$\overline{D}(t) = C\frac{Z_{eff}^2}{\beta^2 a_0^2}\ln\frac{a_0^2 - t^2 + \epsilon + \sqrt{a_0^4 - 2a_0^2(t^2 - \epsilon^2) + (t^2 + \epsilon^2)^2}}{2\epsilon^2} \quad \text{for } t \leq T_{max} + a_0 \quad (4)$$

By assuming that secondary electrons can intersect the sensitive elements and the center of sensitive element is within the distance $T_{max} + a_0$, then

$$\overline{D}(t) = 0 \text{ for } t > T_{max} + a_0 \quad (5)$$

From Eq.(4), it can be seen that the $\overline{D}(t)$ varies with t as $1/t^2$ when $t >> a_0$. The calculated results of $\overline{D}(t)$ varies with t as shown in Fig.1, where $\overline{D}(t)\beta^2a_0^2/Z_{eff}^2$ is plotted against t/a_0 for β=0.05, 0.1 and 0.9, and for $a_0 = 10^{-4}$ and 10^{-6} cm. It can be seen that $D(t)$ has a plateau for $t < a_0$, and when $t > a_0$, $\overline{D}(t)$ decline rapidly with the increment of t and then varies with t as $1/t^2$ until $t = T_{max} + a_0$ where $\overline{D}(t)$ is sudden cut-off. Fig.1 also shows the value of the plateau increases with the a_0. It is easy to see from Eq.(1) that the values of $a_0^2\overline{D}(t)$ increase with an increase of a_0. The value of the plateau is proportional to $Z_{eff}^2\beta^{-2}a_0^{-2}\ln(a_0/\epsilon)$. Calculations show that the mean dose $\overline{D}(t)$ is declining about 10% from t=0 to t=0.9a_0, and 50% at $t = a_0$, and $\overline{D}(1.1a_0)$ is 4 times smaller than the $\overline{D}(a_0)$.

3 Results

Based on Katz's track structure theory[4,5], the inactivation cross sections for V-97 Chinese hamster cells have been calculated by

$$\sigma_{in} = \int_0^\infty 2\pi t(1 - \exp(-\overline{D}(t)/D_0))^m \mathrm{d}t \quad (6)$$

It is interesting that the thindown effect of the ion's inactivation cross sections first increases and then decreases with an increase of LET in the thindown region. Using above equation, the inactivation cross sections for ions of He, C, O, F, Ar, Fe, Ni, Kr, Xe and U bombardment on V-97 Chinese hamster cells are recalculated. The parameters are set to $m = 3$, $a_0 = 0.7\mu m$ and D_0=1.82 Gy. The calculated cross sections are multiplied by a factor of 20, as displayed in Fig.2. The location of hooks along the LET axis is in agreement with experimental results[10] without further adjustment due to the ejecting angle distribution of secondary electrons being considered. The results are the same as the results of Zhang *et al.*[5] which are got by using rather complex radial distribution of dose $D(t)$ and numerical integration methods.

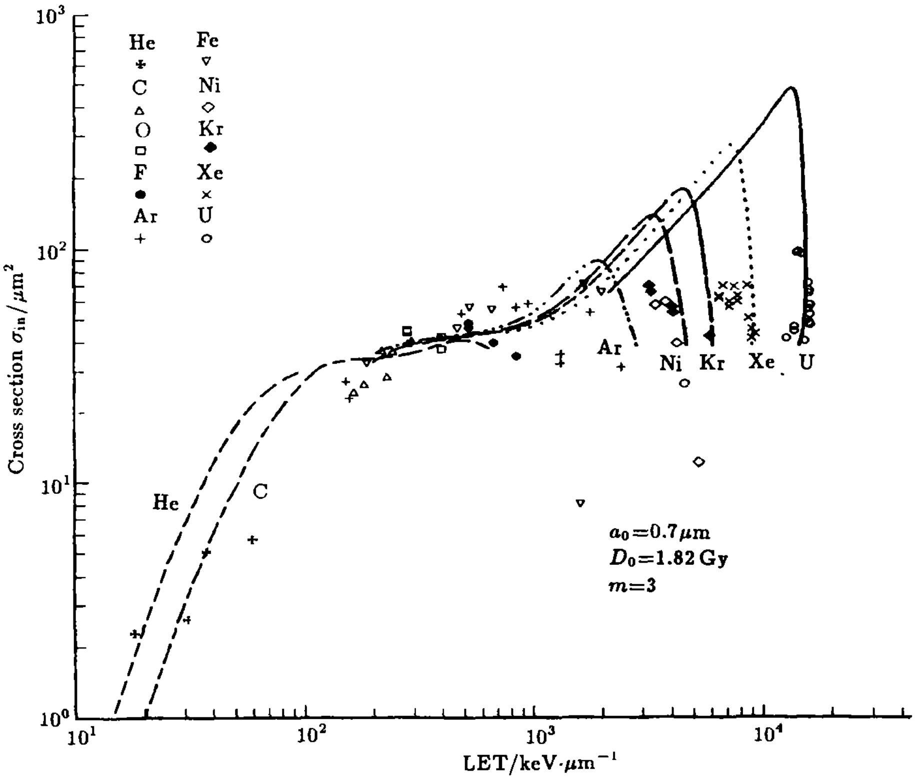

Fig.2 Inactivation cross section vs LET for V-97 Chinese hamster cells. Experimental data are plotted as points while theoretical calculations are plotted as curves

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