# Two-particle correlation and finite size effects of rapidity bin in ultra-relativistic heavy-ion collisions\*

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Abstract Using an aspect of rapidity bins with finite sizes the short-range and longrange behaviors of two-particle correlation, the connection of two particle correlation function with factorial moment and factorial correlator are investigated; and experimental data for <sup>16</sup>O+Em interactions at 14.6, 60 and 200 A GeV and <sup>32</sup>S+<sup>197</sup>Au interactions at 200A GeV are analyzed. It is shown that the bin-average of two-particle correlation function can naturally give a coordinate description of dynamical fluctuation such as the second order factorial moment  $\langle F_2 \rangle$  and 1+1 order factorial correlator  $\langle F_{1,1} \rangle$ .

Keywords Ultrarelativistic heavy ion collision, Two particle correlation, Fluctuation

# 1 Introduction

The dynamical behavior of the fluctuation of multihadron production in high energy heavy ion collisions has been being the focus<sup> $[1 \sim 11]</sup>$  of</sup> keen interest due to the rising resolution of experimental measurement and the available precise new data. In the description of hadronic final states with large multiplicity created in hadron-hadron or nucleus-nucleus collisions at high energies, an important analysis on the reaction mechanism is in the longitudinal phase space as measured by pseudorapidity variable  $\eta = -\ln \tan \theta / 2$  in experiments<sup>[12]</sup>. Typically, event histogram is highly irregular. Bialas and Peschanski put forward a scaled factorial moment method<sup>[1,2]</sup> to detect such behavior, which is usually referred to as "intermittency" in multihadron physics. Experimental data in a varietv of collision processes  $[3 \sim 10]$  indeed show an approximately power dependence of scaled factorial moment on rapidity bin for its decreasing size only in a typical interval of size of bin from 1 to 0.1. It has been suggested that a quarkgluon plasma phase transition could give rise to such behavior<sup>[11]</sup>. Several authors have proposed another ansatz<sup>[13,14]</sup> that the measured effects within present experimental resolution, can be understood from two-particle correlation.

In this paper, we start right in with definition of two-particle correlation function to

give out experimental observation through lattice procedure for rapidity space. With such a definition, we observed finite size effects of rapidity bin of two-particle correlation behavior. Under the approaches, it is natural to get the uniform description for  $\langle F_2 \rangle$  and  $\langle F_{1,1} \rangle$ , without any additional assumption and approximation.

# 2 Definition of correlation function

The inclusive rapidity density functions of single- and two-particles are defined by

$$\rho_1(y) = \langle \sum_i \delta(y - y_i) \rangle \tag{1}$$

$$\rho_2(y_1, y_2) = \langle \sum_{i,j} \delta(y_1 - y_i) \delta(y_2 - y_j) \rangle \quad (2)$$

where sum of *i* and *j* is over all produced particles in one event and the angular brackets  $< \cdots >$  denote the events ensemble average. The normalization conditions for  $\rho_1(y)$  and  $\rho_2(y_1, y_2)$  in the considered rapidity domain  $\Omega$ are defined as

$$\int_{\Omega} \rho_1(y) dy = < n >_{\Omega} \tag{3}$$

and

$$\int_{\Omega} \rho_2(y_1, y_2) dy_1 dy_2 = < n(n-1) >_{\Omega}$$
 (4)

\*The Project Supported by the National Natural Science Foundation of China, and the Natural Science Foundation of Hubei Province

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where  $\langle n \rangle_{\Omega}$  and  $\langle n(n-1) \rangle_{\Omega}$  are mean multiplicity and the second factorial moment of produced particles in  $\Omega$ , respectively. The twoparticle correlation function by deducing the influence from the rapidity density distribution of single-particle is

$$r_2(y_1, y_2) = \frac{\rho_2(y_1, y_2)}{\rho_1(y_1)\rho_1(y_2)} \tag{5}$$

Now, let us analyze the situation of the lattice procedure of the considered rapidity domain  $\Omega$  which is divided into M bins with size  $\omega = \Omega/M$ . It is obvious that the single- and two-particle rapidity density functions for such a system with finite size of bin and finite number of bins can be transformed from the theoretical continuous case with infinitesimal  $\omega$  and infinite M to experimental discrete case

$$\rho_1(y) \to \rho_1^{(\omega)}(m) = \frac{\langle n_m \rangle_\omega}{\omega}, m = 1, 2, \cdots, M$$
(6)

$$\rho_2(y_1, y_2) \to \rho_2^{(\omega)}(m_1, m_2) = \begin{cases} < n_m(n_m - 1) >_{\omega} / \omega^2 & \text{for } m_1 = m_2 = m, \\ < n_{m_1} n_{m_2} >_{\omega} / \omega^2 & \text{for } m_1 \neq m_2. \end{cases}$$
(7)

Their normalization conditions in the discrete case can be rewritten as

$$\frac{1}{M}\sum_{m=1}^{M}\rho_{1}^{(\omega)}(m)\omega = \frac{\langle n \rangle_{\Omega}}{M}$$
(8)

$$\frac{1}{M^2} \sum_{m_1, m_2=1}^{M} \rho_2^{(\omega)}(m_1, m_2) \omega^2 = \frac{1}{M^2} \sum_{m=1}^{M} \langle n_m(n_m - 1) \rangle_\omega + \frac{1}{M^2} \sum_{m_1 \neq m_2, 1}^{M} \langle n_{m_1} n_{m_2} \rangle_\omega$$
(9)

So the two-particle correlation function defined as Eq.(5) in the discrete case should be

$$r_{2}(y_{1}, y_{2}) \rightarrow r_{2}^{(\omega)}(m_{1}, m_{2}) = \begin{cases} < n_{m}(n_{m} - 1) >_{\omega} / < n_{m} >_{\omega}^{2} & \text{for } m_{1} = m_{2} = m \\ < n_{m_{1}}n_{m_{2}} >_{\omega} / < n_{m_{1}} >_{\omega} < n_{m_{2}} >_{\omega} & \text{for } m_{1} \neq m_{2} \end{cases}$$
(10)

As known that the second scaled factorial moment and the 1+1 order scaled factorial correlator are defined as

$$\langle F_2^{(\omega)} \rangle = \frac{1}{M} \sum_{m=1}^M \frac{\langle n_m(n_m-1) \rangle_\omega}{\langle n_m \rangle_\omega^2} \tag{11}$$

$$\langle F_{1,1}^{(\omega)}(K)\rangle = \frac{1}{(M-K)} \sum_{m=1}^{M-K} \frac{\langle n_m n_{m+K} \rangle_{\omega}}{\langle n_m \rangle_{\omega} \langle n_{m+K} \rangle_{\omega}} \quad K = 1, 2, \cdots, M-1$$
(12)

Hence we have

$$\overline{r_2^{(\omega)}(K)} = \frac{1}{M-K} \sum_{m=1}^{M-K} r_2^{(\omega)}(m, m+K) = \begin{cases} \langle F_2^{(\omega)} \rangle & \text{for } K = 0\\ \langle F_{1,1}^{(\omega)}(K) \rangle & \text{for } K > 0 \end{cases}$$
(13)

An important point can be drown out that the bin-average of the two-particle correlation function in the discrete case for the rapidity space is not else, but the direct universal measure of the second scaled factorial moment and the 1+1 order scaled factorial correlators under a coordinate description.

# 3 Data analysis

To show the feasibility of the above method and to give some information about two particle correlation in relativistic energy heavy ion collision experiments, some available data from EMU01 experiment<sup>[15]</sup> at CERN/SPS and BNL/AGS are analysed.

The present analysis focuses on minim bias samples of  ${}^{16}\text{O}+\text{Em}$  (Nuclear Emulsion) and central collisions for  ${}^{32}\text{S}+\text{Au}$  which means that essentially all projectile nucleons participate in interactions. The pseudorapidity ranges  $\Omega$  are 0.8-2.8, 1.4-3.4 and 2.0-4.0 for 14.6, 60 and 200*A* GeV respectively. The region  $\Omega$  is divided into *M* bins of unit with  $\omega = \Omega/M$ . *M* is 2, 4, 8, 16 and 20 for  ${}^{16}\text{O}+\text{Em}$  data, and up to 40 for <sup>32</sup>S+Au samples. Correlation gap is defined as  $D = K\omega$ , allowing K being 0, 1, 2, ..., M - 1. Fig.1 shows the results of two-particle correlation for <sup>16</sup>O+ Em interactions. Fig.2 is about results of <sup>32</sup>S + Au central collisions. In the Fig.1.2. it can be understood that  $r_2^{(\omega)}$ 



Fig.1 Averaged two particle correlation for <sup>16</sup>O+Em interaction at 14.6, 60 and 200 A GeV,  $\overline{\tau_2}$  corresponding to  $\langle F_2^{\omega} \rangle$  for K = 0,  $\overline{\tau_2}$  to  $\langle F_{1,1} \rangle$ for K > 0, a),b),c),d) and e) corresponding to bin partition M=2, 4, 8, 16 and 20, respectively

is equivalent to  $\langle F_2 \rangle$  for K = 0, and to  $\langle F_{1,1} \rangle$ for K > 0. The values of  $\overline{r_2^{(\omega)}}$  decrease with the K from 0 to (M-1), corresponding to the gap D between different bins, and increase very slowly with increasing M, i.e. with decreasing  $\omega$ . Correlation raises with increasing beam energy, and decreases rapidly with increasing mass of projectile and target nuclei. Qualitatively, the strong correlations in small rapid-



Fig.2 Averaged two particle correlation for  ${}^{32}S + Au$  interaction at 200 A GeV,  $\overline{\tau_2}$  corresponding to  $\langle F_2^{(\omega)} \rangle$  for K = 0,  $\overline{\tau_2}$  to  $\langle F_{1,1} \rangle$  for K > 0, a),b),c),d),e) and f) corresponding to bin partition M=2, 4, 8, 16, 20 and 40, respectively

Solid points and open circles with error bar correspond to data points, solid lines correspond to fitting values of model with exponential-type

ity bins probably result from resonance decays and local charged compensation which probably means that production particles in high energy heavy ion collision processes much more prefer to occurring in pairs, and the correlation between two particles under big gap D implies long-range correlation which mainly comes from energy-momentum conservation constraints.

When 
$$M \rightarrow \infty$$
 (i.e.  $\omega \rightarrow 0$ ), two-

particle correlation function could be simply Reparametrized as

$$r_2(y_1, y_2) = \alpha \cdot \exp(-\frac{|y_2 - y_1|}{\beta})$$
 (14)

where  $\alpha$  is the correlation strength and  $\beta$ the corresponding length. One can calculate the corresponding functions in discrete case as

Refs.[13,14]

It is easy to see that bin-average of twoparticle correlation functions in discrete case with finite size of  $\omega$  for K = 0,  $r_2^{(\omega)}(K=0)$ , i.e.  $\langle F_2 \rangle$ , can be rewritten as the integration result of two-particle correlation function in infinite case of  $\omega \to 0$  by using precise bin-wise method<sup>[13,14]</sup>,

$$\overline{r_2^{(\omega)}(K=0)} = 1 + \frac{2\alpha\beta}{\omega^2} \left[ \omega - \beta(1 - \exp(-\frac{\omega}{\beta})) \right]$$
(15)

and the values  $r_2^{(\omega)}(K)$  with  $K \ge 0$ , i.e. the 1+1 order factorial correlator  $\langle F_{1,1}(D) \rangle$ ,  $D = K\omega$ , can be rewritten as following

$$\overline{\frac{\lambda^{(\omega)}}{2}(K)} = 1 + \frac{2\alpha\beta^2}{\omega^2} \exp(-\frac{K\omega}{\beta}) \left[\cosh(\frac{\omega}{\beta} - 1)\right]$$
(16)

Using above two expressions to fit the observed data for  ${}^{32}S+Au$  central collisions at 200 A GeV, the uniform parameters  $\alpha \sim 0.107$ and  $\beta \sim 8.45$  are available. The histograms in Fig.2 correspond to the fitting results. The parametrization results show that two-particle correlation functions of exponential-type can describe the data well.

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## 4 Conclusion

Two-particle correlation behavior reveals not only density fluctuation features in local phase space, but also correlation characters caused by density fluctuation between different bins.  $r_2^{(\omega)}(K)$  decreases with increasing gap  $D = K\omega$  and increasing mass of projectile and target nuclei, and increases with increasing beam energy. Bin-averaged correlation function gives out the coordinate descriptions for factorial moment  $F_2$  and correlators  $F_{1,1}$ . Two-particle correlation function of exponential- type can describe the experimental data of  ${}^{32}S$ +Au central collisions at 200 A GeV.

### Acknowledgements

Authors are very grateful to Lund University (Sweden) for their kind help

#### References

1 Bialas A, Peschanski R. Nucl Phys, 1986, 272B:703

- 2 Bialas A, Peschansko R. Nucl Phys, 1988, 308B:857
- 3 Adamovich M I, Aggarwal M M, Alexandrov Y A et al. (EMU01). Phys Rev Lett, 1990, 65:412
- 4 Adamovich M I, Aggarwal M M, Alexandrov Y A et al. (EMU01). Accepted by Z Phys C, 1997
- 5 Braunschweig W, Gerhards R, Kirschfink F J et al. (TASSO). Phys Lett, 1989, 231B:548
- 6 Behrend H J Criegee L, Field J H et al. (CELLO). Phys Lett, 1991, 256B:97
- 7 Abren P et al. (DELPHI). Phys Lett, 1990, 247B:137
- 8 Agababyan N M, Böttcher H, Botterweck F et al. (NA22). Phys Lett, 1991, 261B:165
- 9 Albajar C, Albrow M G, Allkofer O C et al. (UA1). Nucl Phys, 1990, 345B:1
- 10 Wu Yuan-Fang, Liu Lian-Shou. Phys Rev Lett 1993, 70:3197
- 11 Hwa R C. Acta Physica Polonaca, 1996, 27B:1789 and references therein
- 12 Cai Xu, Huang Hong, Liu Lian-Shou et al. Nucl Techni (in Chinese), 1990, 13:733
- 13 Carruthers P, Sarcevic Ina. Phys Rev Lett, 1989, 63:1562
- 14 Elze H T, Sarcevic Ina. Phys Rev Lett, 1992, 68:1988
- 15 Cai Xu et al. In: Stenlund E and Otterlund I eds. Proceedings from 12 th EMU01 Collaboration Meetings, Lund, 1993