# Analysis of the negative-parity yrast band in $^{155}$Tb

WU Xing-Ju[1,2], XU Jin-Zhang[2], CHEN Xing-Qu[2,3]

(*Center of Theoretical Nuclear Physics, National Laboratory of Heavy Ion Accelerator, Lanzhou 730000;*

[1] *Physics Department, Lu-an Teacher's College, Lu-an, Anhui 237012;*

[2] *Department of Modern Physics, Lanzhou University, Lanzhou 730000;*

[3] *Shanghai Institute of Nuclear Research, the Chinese Academy of Sciences, Shanghai 201800*)

**Abstract** Using the particle-rotor model, the energy spectrum and the $B(M1)/B(E2)$ ratio of the negative-parity yrast band in $^{155}$Tb are investigated and compared with the experimental data before and after the band-crossing. It is noted that before and after the band-crossing the moment of inertia of the core is a smooth function of the total angular momentum $I$ and can be described by the ab formula. The yrast band of $^{155}$Tb with negative parity may be triaxially deformed ($\gamma$=$-13°$) before and an axially symmetric shape ($\gamma$=$0°$) after the band-crossing.

**Keywords** High spin nuclear states, The ab formula

**CLC numbers** O571.21+1, O571.22, O571.24

## 1 Introduction

Many theoretical and experimental studies have been done recently on high-spin states of odd-Z rare-earth nuclei. Among them the $N$=90 nuclei dwell within a highly transitional area of deformation space. These nuclei are especially susceptible to shape driving forces by various competing processes.

Rotation of deformed nuclei is an intricate topic of nuclear structure, and high spin states of rare-earth nuclei provide many interesting data to test collective nuclear models. For years, the ideal rigid rotor model was used as the approximation to the zeroth order for rotation motion, but the deviation of its rotational energy spectrum formula, $E(I) = AI(I+1)$, from the observed data of rotational bands had been found at different nuclear regions to a different extent. A much better formula of only two parameters for rotational energy spectrum was deduced phenomenologically[1] and derived theoretically from a Bohr Hamiltonian[2] and is now often called the ab formula:

$$E(I) = a[\sqrt{1+bI(I+1)} - 1] = \frac{\hbar^2}{2J_0(I)}I(I+1) \quad (1)$$

in which is included also the idea of a variable moment of inertia $J_0(I)$:

$$J_0(I) = J_{00}\frac{1+\sqrt{1+bI(I+1)}}{2} \quad (2)$$

This ab formula describes satisfactorily all rotational spectra of ground-state bands of the normal deformed even-even nuclei in rare-earth and actinide regions[2,3], and also describes properly the super-bands ($s$-bands)[4]. It has been proved recently that the

Supported by the National Natural Science Foundation of China under Grant No. 19975023, and by the Science Foundation of Nuclear Industry of China under Grant No. Y7197AY103

*Manuscript received date: 1999-11-11*

transition energy spectra of superdeformation bands can also be excellently reproduced by this ab formula, Eq.(1), and the moment of inertia $J_0$, Eq.(2), derived from Eq.(1)[5~8].

Very recently, a new high-spin states result for $^{155}$Tb was reported[9]. We note that the $B(M1)/B(E2)$ transition strength ratios were extracted from the data and comparisons were made with geometrical model[10]. Their theoretical $B(M1)/B(E2)$ ratios overestimated the signature splitting by an order of magnitude in the one-quasiparticle region. This deficiency may be caused by the assumption of an axially symmetric shape. Is it triaxially deformed?

Here we shall calculate both the energy spectra and transition probabilities of the negative-parity yrast band in $^{155}$Tb using the particle-rotor model, and compare our results with experimental data in order to study the question about the triaxiality of the negative-parity yrast band in $^{155}$Tb before and after the band-crossing, and at the same time to test the applicability of Eqs.(1) and (2) to the yrast $s$-band after the band-crossing region.

## 2 The model

Based on the experimental level scheme, the last proton occupies [532]5/2 orbital in negative parity yrast states of $^{155}$Tb, which is not mixed up with neighboring orbitals of positive parity. Consequently, the single-j particle-triaxial-rotor model[11,12] is used to calculate both the energy spectra and transition probabilities before and after the band-crossing. There is only one quasiproton in the $h_{11/2}$ subshell coupled with the even-even core before band-crossing, whereas two $i_{13/2}$ neutrons are aligned by the rotation after band-crossing, thus three quasiparticles are coupling with the core. If the last proton angular momentum is expressed as $\vec{j}_p$, the neutron spin alignment as $\vec{J}_n$, and the collective angular momentum of the even-even core as $\vec{R}$, then the total angular momentum $\vec{I} = \vec{R} + \vec{J}_n + \vec{j}_p$. To deal with $s$-band after band-crossing, our total Hamiltonian is written as a sum of the rotor part and the intrinsic part for the quasiproton and quasineutrons:

$$H = H_{\rm rot} + H_{\rm intr,p} + H_{\rm intr,n} \tag{3}$$

Here triaxial-rotor Hamiltonian for the core is

$$H_{\rm rot} = \sum_{k=1}^{3} \frac{\hbar^2}{2J_k} R_k^2 = \sum_{k=1}^{3} \frac{\hbar^2}{2J_k}[(I_k - J_{nk}) - j_{pk}]^2 \tag{4}$$

in which $J_k$ is the moment of inertia associated with rotation about the intrinsic $k$th axis. In numerical calculations, we take the moment of inertia of hydrodynamical type

$$J_k = \frac{4}{3} J_0(I) \sin^2(\gamma + k\frac{2\pi}{3}), \qquad k = 1,2,3 \tag{5}$$

in which $J_0(I)$ is a smooth function of the total angular momentum $I$ and is expressed in Eq.(2), wherein parameters $J_{00}$ and $b$ are estimated by fitting $\gamma$-transition energies.

It is not practical to perform an exact particle-rotor model calculation with three quasiparticles for a triaxial shape because of the large configuration space. Due to the one-particle character of the electromagnetic transition operators, the change in the number of quasiparticles does not have any influence on the signature-dependent part of the transition-matrix elements. We can freeze the intrinsic degrees of freedom of the aligned neutrons, just as the simplified model proposed by Ref.[11] did to deal with the $s$-band configuration. Since at present we are specially interested in the spin region of the $s$-band in which the neutron spin alignment is nearly constant, viz., $J_n \approx 10$, then the contribution from the intrinsic Hamiltonian of the aligned neutrons is an additional constant, and this term $H_{\rm intr,n}$ in Eq.(3) can be ignored as done in Refs.[4,11]. We can also assume that $\vec{J}_n$, the neutron spin alignment of the $s$-band, is parallel with $\vec{R}$, the collective angular momentum of the (triaxial) core, and then an angular momentum for the $s$-band, $\vec{X}$, can be defined as in Refs.[4,11]:

$$\vec{X} = \vec{I} - \vec{J}_n = \vec{R} + \vec{j}_p \tag{6}$$

The coupling scheme of angular momenta in s-band is shown in Fig.1.

Thus the three-quasiparticle problem of the total Hamiltonian (3) of the $s$-band is reduced to a one-quasiparticle problem quite similar to that of the usual $g$-band:

$$H = \sum_{k=1}^{3} \frac{\hbar^2}{2J_k} R_k^2 + H_{\rm intr,p} \tag{7}$$

which is of the usual form of one- quasi-particle configuration and is easy to be diagonalized. This Hamiltonian can be used for both $g$-band ($J_n$=0, $\vec{X} = \vec{I}$, $\vec{R} = \vec{I} - \vec{j}_p$) and $s$-band ($J_n \approx 10$, $\vec{X} = \vec{I} - \vec{J}_n$, $\vec{R} = \vec{X} - \vec{j}_p$), The only difference is that the rotor angular momentum of the $s$-band is calculated as $\vec{R} = \vec{X} - \vec{j}_p$. The detailed formulas necessary for the calculation in the particle-rotor model can be found in Refs.[6,11].

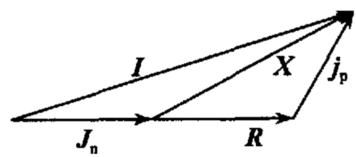


Fig.1 Geometric relation of the coupling angular momenta of the $s$-band

Hamiltonian, Eq.(7), is diagonalized by using the BCS one-quasiparticle states for the particle wave functions. Using the resulted wave functions $B(M1)$ and $B(E2)$ values can be calculated.

## 3 Results and discussion

The calculated energy difference, $E(I) - E(I-1)$, and $B(M1)/B(E2)$ ratio are compared with the measured values in Figs.2 and 3 for [532]5/2 band of $^{155}$Tb, respectively. The model is not applicable to the states involving strong mixing between the $g$-band and the $s$-band. Thus, the states with $31/2 < I < 41/2$ were not calculated here. Since after the band-crossing, the $s$-band is known to consist of $(i_{13/2})^2$ quasineutrons, their alignment $J_n \approx 10$ and the neutron g factor $g_{J_n}$=−0.20 are used in the calculation[11]. From the good agreement of the calculated value with the observed ones we can know the followings.

(1) Comparing the calculated signature splitting of the energies with the observed values, parameters of the particle-rotor model are chosen before and after the band-crossing so that the observed data are, on the average, reproduced as well as possible. Using the same parameters, the calculated ratios of $B(M1;I\to I\text{-}1)$ to $B(E2;I\to I\text{-}2)$ agree quite well with the observed ones and so the adopted parameters are reasonable. The triaxiality $\gamma$ values adopted in our calculation for $\pi[532]5/2^-$ band in $^{155}$Tb, are $\gamma=-13^\circ$ before and $\gamma=0^\circ$ after the band-crossing. The result given in Ref.[9] by using the

geometrical model with an axially symmetric shape can not reproduce the experimental signature dependence of the $B(M1)/B(E2)$ ratios before the band-crossing. Therefore, the nuclear shape may be triaxial. Since $i_{13/2}$ neutrons are predicted to polarize the nucleus towards a symmetric prolate shape, the $\gamma$=0° should be able to describe the observed data after band-crossing. From our good agreement of the calculated values with experimental data, we say that the nuclear shape of the yrast band of $^{155}$Tb with negative parity may be triaxially deformed ($\gamma$=−13°) before and axially symmetric ($\gamma$=0°) after the band-crossing.

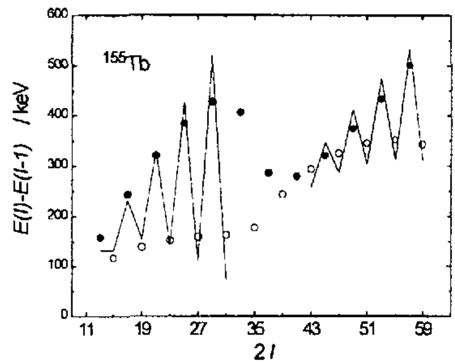


**Fig.2** Energy difference $\Delta E = E(I) - E(I-1)$ versus spin $I$ for the [532]5/2$^-$ band in $^{155}$Tb. The experimental values (taken from Ref.[9]) are shown as solid and open circles, while the calculated values are connected by the solid lines. The solid (open) circle corresponds to the energy difference from the state with the signature $\alpha$=+1/2(−1/2) to $\alpha = -1/2(+1/2)$. The parameters used in the particle-rotor model are $\gamma = -13°$ (0°), $\Delta/\kappa$=0.40(0.40), $\lambda/\kappa$ = 0.0 (0.0), $J_{00}\kappa$=53 (65), $b \times 10^3$ = 7.0 (5.0), and $J_n$=0 (10) before (after) the band-crossing. Here $\kappa$ is used as an energy unit which is determined by normalizing the calculated values of $\gamma$-transition energies to one of the experimental data

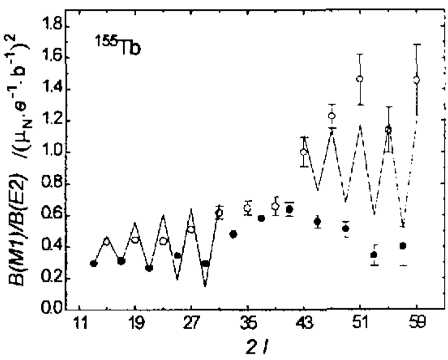


**Fig.3** $B(M1{:}\Delta I{=}1)/B(E2;\ \Delta I{=}2)$ ratios versus spin $I$ for the [532]5/2$^-$ band in $^{155}$Tb. Experimental values with error bars (taken from Ref.[9]) are shown as solid (open) circles, corresponding to the energy difference from the state with the signature $\alpha = +1/2(-1/2)$ to $\alpha$=−1/2(+1/2). The calculated values are connected by the solid lines. The parameters are intrinsic quadrupole moment squared $Q_0^2 = 30e^2b^2$, effective electric charge $e_{\rm eff} < j \mid r^2 \mid j > /Q_0 = 0.20e$, $g_l$=1.0, $g_s$=3.91, $g_{\rm R}$=0.30, and $g_{J_n}$=−0.20. Other parameters of the particle-rotor model are the same as those used in Fig.2

(2) The good agreement of the calculated values with the experimental data also shows that the simplified model[11], Eq.(7), dealing with $s$-band configuration is a good approximation. The angular momenta of the two rotation-aligned quasineutrons $\vec{J_n}$ can be assumed constant ($J_n$ ≈10 in the $s$-band), and in parallel with $\vec{R}$, the collective

angular momentum of the core. Then we can freeze the degrees of freedom of two aligned quasineutrons and neglect their intrinsic term in the total Hamiltonian, thus the 3qp (quasiparticle) problem, Eq.(3), is reduced to a 1qp problem, Eq.(7).

(3) The calculated energy spectra coincide with the observed data. It shows that the variation of the moment of inertia of the even-even core, $J_0(I)$, against angular momentum $I$ is well described by Eq.(2) both before and after band-crossing, but with different $J_{00}$ and $b$ values, because the two rotational bands are different. For a given fixed $\gamma$-value the moment of inertia $J_0(I)$ is a smoothly-increasing function of $I$, which is reasonable since $I$-dependence comes mainly from the decrease of the neutron pairing correlation as $I$ increases. The $I$-dependence of the moment of inertia of the core rotation in the expression (2) is deduced from the ab formula. Therefore, the ab formula, Eq.(1), can be used to depict not only the $g$-band but also $s$-band.

## References


1 Holmberg P, Lipas P O. Nucl Phys, 1968, **A117**:552-560
2 Wu C S, Zeng J Y. Commum Theor Phys, 1987, **8**:51-74
3 Xu F X, Wu C S, Zeng J Y. Phys Rev, 1989, **C40**:2337-2341
4 Liang Y S, Wu X J, Xu J Z *et al.* Phys Rev, 1999, **C60**:054305
5 Wu C S, Zeng J Y, Xing Z *et al.* Phys Rev, 1992, **C45**:261-274
6 Chen X Q, Xing Z. J Phys, 1993, **G19**(11):1869-1877
7 Xing Z, Wang Z X, Chen X Q. Chin Phys Lett, 1998, **15**:170-172
8 Xing Z, Wang Z X, Chen X Q *et al.* High Energy Phys Nucl Phys (in Chinese), 1998, **22**(6):545-549
9 Hartley D J, Brown T B, Kondev F G *et al.* Phys Rev, 1998, **C58**(5):2720-2735
10 Janzen V P, Liu Z M, Carpenter M P *et al.* Phys Rev, 1992, **C45**:613-639
11 Hamamoto I, Mottelson B R. Phys Lett, 1986, **B167**(4):370-374
12 Hamamoto I, Sagawa H. Phys Lett, 1988, **B201**(4):415-419