Coherent transition radiation from REB in plasma ripple

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Abstract The coherent transition-radiation emission from an underdense relativistic beam of electrons, travelling through a dense plasma ripple, was studied. The evolution of this radiation field is described by a set of self-consistent pendulum-wave equations. Analytic calculations of the small-signal gain and numerical computations of the nonlinear saturation of this emission are presented. It is shown that such a device may provide a source of tunable coherent radiation ranging from the microwave to the infrared region.

Keywords Transition radiation, Free electron laser, Plasma

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1 Introduction

In a free-electron laser (FEL), a relativistic electron beam (REB) passing through a periodic transverse magnetic field in an undulator generates coherent radiation ranging from the infrared to the ultraviolet region[1,2]. However, the undulator period of the presently existing FEL is in the order of cm and is limited by the practical difficulty of placing very strong and very small permanent magnets or electromagnets close together in an alternating array. This implies that to achieve a short-wavelength radiation the conventional FEL would require a high energy REB, but operating with low efficiency. In recent years, there has been an increased interest in short-period undulators. One concept is the ion-ripple laser (IRL)[3~8], which promises to be a very short-period undulator. The ion-ripple laser makes use of a REB obliquely passing through an underdense plasma ripple. As the REB density n_b is equal to or higher than the plasma density n_p, the plasma electrons are all expelled out of the REB path and an ion-ripple channel is created. The ion-ripple field is experienced as a stationary undulating force by the relativistic electron. Subjected to the transverse electrostatic field of the ion ripple, the relativistic electrons execute transverse oscillations and emit radiation. Because of the promising of the shorter undulator period and strong undulator force, the IRL appears superior to the conventional FEL.

Probably one of the main experimental challenges of the IRL is the maintenance of the ion ripple after the plasma electrons are not available to neutralize the ion's space charge. The effective relaxation time (the ion-ripple modulation factor, α_{IR}, is in the order of unit in the beginning, and to be decreased by one order of magnitude after

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time τ_r) is approximately given by[8] $\tau_r \sim 2(\pi\mu)^{1/2}/\omega_{pe}$, where $\omega_{pe} = (4\pi n_0 e^2/m_e)^{1/2}$ is the plasma frequency, and $\mu = m_e/m_i$ is the mass ratio (n_0 is the average density of the plasma ripple, e is the charge of electron, and m_e and m_i are the rest mass of the electron and the ion, respectively). For instance, the effective relaxation time $\tau_r \sim 1$ ns, for a plasma of moderate densities $n_0 \sim 10^{14}\,\text{cm}^{-3}$, and $\mu \sim 10^{-5}$. Moreover, the IRL generally requires that the duration τ_p of the REB is much longer than the time of expulsion for the REB τ_e. The time of expulsion for the REB of radius R can be estimated by $\tau_e \sim 10R/c$[8], where c is the velocity of light. For instance, the time of expulsion is $\tau_e \sim 33\,\text{ps}$ for a REB with radius 1 mm.

In this work, we consider an alternative case, in which an underdense relativistic electron beam passes through a dense plasma ripple. Clearly, as $n_b \ll n_p$, only small partial plasma electrons are expelled out of the REB column, and an ion ripple channel can not be created. In this case, the time during the plasma ripple is maintained and is much longer than that for ion ripple since the electrons of the plasma ripple are available to neutralize the ion's space charge. Thus, the plasma ripple serves as a stationary periodically modulated dielectric medium. It has been known that a relativistic electron may emit coherent transition radiation, as it passes through a periodically modulated dielectric medium[9,10]. The fast oscillation of the relativistic electron's phase, which is induced by the periodically modulated medium, is a key factor to the process of the stimulated transition-radiation emission. A laser device based on this mechanism may be called as plasma-ripple transition- radiation laser (PRL).

2 Basic equations

For simplicity, we neglect the transverse variation of the plasma density and take the density of the plasma ripple to be

$$n_p = n_0[1 + \alpha_R \cdot \cos(\vec{k}_R \cdot \vec{r})] \tag{1}$$

where α_R is the plasma-ripple modulation factor and k_R the wave number of the plasma ripple. As an electromagnetic wave propagates through such a plasma ripple, it will experience a periodically modulated dielectric medium. The dielectric constant of the plasma ripple can be expressed by

$$\varepsilon = \varepsilon_0 - (\alpha_R \omega_{pe}^2/\omega^2)\cos(\vec{k}_R \cdot \vec{r}) \tag{2}$$

with $\varepsilon_0 = 1 - (\omega_{pe}/\omega)^2$, where ω is the radiation frequency.

We consider a one-dimensional model, in which a REB obliquely passes through the plasma ripple at a small angle θ with respect to the direction of the plasma ripple and radiation field undergoes spatial amplification as it interacts with a REB through the plasma ripple. The radiation field is taken to be a plane wave with slowly varying amplitude E_s and phase φ, propagating through the plasma along the direction of plasma ripple, z axis. For simplicity, we assume that $\alpha_R(\omega_{pe}/\omega)^2 \ll \varepsilon_0$, and the radiation field is independent of the transverse position x and y. With these simplifications, the vector potential of the radiation field may be expressed by

$$A = \frac{m_e c^3 E_s}{e\omega} \sin\psi \vec{x} \tag{3}$$

where the radiation phase ψ is defined as

$$\psi = \int_o^z k dz - \omega t + \varphi \tag{4}$$

with $k^2 = \omega^2 \varepsilon / c^2$. As a relativistic electron passes through the plasma ripple along the z direction and interacts with the radiation field, it will experience an oscillating phase. One may express the relativistic electron's phase as a fast oscillating term and a slowly varying (or constant) term, which are given by the following form

$$\psi = \overline{\psi} - \sigma \sin(k_R z) \tag{5}$$

where the slowly varying phase term and the strength factor are respectively given by $\overline{\psi} = (k_0 - \omega/\beta_z c)z + \varphi$ and $\sigma = \alpha_R \omega_{pe}^2 / 2\omega\varepsilon_0 k_R c$, with $k_0^2 c^2 = \omega^2 - \omega_{pe}^2$, $\beta_z = 1 - 1/2\gamma_z^2$ (the z-component velocity of relativistic electron), and $\gamma_z^2 = \gamma^2/(1 + \gamma^2\theta^2)$ (γ is the Lorentz factor of the relativistic electron). Since the electron bunching on a optical wavelength scale is the key factor to a stimulated emission process, the fast oscillation of the relativistic electron's phase may cause emission into all harmonics, e.g., n=1, 2, 3,$\cdots$. This process is similar to that for the harmonic generation in a planar undulator FEL[11].

To study the interaction between the radiation field and the REB, we firstly analyze the radiation frequency of the PRL. Similar to that of the conventional FEL[11], the resonant condition of the nth harmonic emission in the PRL is given by $\omega = (nk_R + k_0)\beta_z c$. Combining the dispersion relation of radiation in plasma and the resonant condition, one obtains, for $\omega \geq \gamma_z n k_R \beta_z c$, the expression for the resonant frequency of the nth harmonic

$$\omega_n = \frac{\omega_{ns}^R}{1 + \beta_z} (1 \pm \beta_z \sqrt{1 - X_n}) \tag{6}$$

with wave number $k_n = (\omega_{ns}^R / c)(\beta_z \pm \sqrt{1 - X_n})$, where the resonant frequency ω_{ns}^R in vacuum is given by $\omega_{ns}^R = nk_R \beta_z c/(1 - \beta_z)$ and the factor $X_n = (\omega_{pe}/\gamma_z k_R \beta_z c)^2$ for $0 \leq X_n \leq 1$. The radiation frequency exhibits two branches, a high-frequency branch ω_n^+ corresponding to the positive sign and a lower-frequency branch ω_n^- corresponding to the negative sign. For a high relativistic electron beam ($\gamma \gg 1$), one may rewrite the two resonant frequencies of the nth harmonic as

$$\omega_n^+ \cong \kappa_n \omega_n^- \cong \kappa_n \omega_{ns}^R / (1 + \kappa_n) \tag{7}$$

where $\kappa_n \equiv \omega_n^+ / \omega_n^- = (1 + \beta_z \sqrt{1 - X_n})/(1 - \beta_z \sqrt{1 - X_n})$ is the frequency ratio parameter of the nth harmonic, with $1 \leq \kappa_n \leq (1 + \beta_z)/(1 - \beta_z)$.

Following the usual methods developed to determine the harmonic emission in conventional FEL[11], one may obtain a set of self-consistent pendulum-wave equations to describe the evolution of the nth harmonic radiation field in PRL. The results may be expressed by

$$\frac{d^2}{d\overline{z}_n^2}\xi_n = -(a_n \exp(i\xi_n) + c.c) \tag{8}$$

$$\frac{d}{d\overline{z}_n}a_n = \langle \exp(-i\xi_n)\rangle \tag{9}$$

where the relativistic electron's ponderomotive phase is $\xi_n = (k + nk_R - \omega/\beta_z c)z$, the scaled position is $\overline{z}_n = z/L_{Gn}$, the scaled gain length of the nth harmonic emission is defined as $L_{Gn} = (1/2k_R\rho_n)(\omega_{ns}^R/\omega_n)$, the mode-dependent PRL parameter is given by $\rho_n = \rho_0[nJ_{-n}^2(n\sigma)]^{1/3}(\omega_{ns}^R/\omega_n)^{2/3}$ and the scaled optical amplitude is $a_n = \sqrt{n}E_s\exp(i\varphi)/\sqrt{4\pi m_e c^2\gamma n_b \rho_n(1+\gamma^2\theta^2)}$. Here J_n is the Bessel function of the first kind and nth order and ρ_0 is the fundamental PRL parameter, which is given by $\rho_0 = \gamma_0^{-1}(\gamma_0\theta\omega_b/4k_R c)^{2/3}$ with plasma frequency of REB $\omega_b = (4\pi n_b e^2/m_e)^{1/2}$ and n_b being the density of REB. Clearly, the coupling between the relativistic electrons and the harmonic radiation is now characterized by the Bessel function $F_n = J_{-n}(n\sigma)$, which represents the effects of the fast oscillation of the relativistic electron's phase on the stimulated emission process.

3 Analytical results and numerical examples

These self-consistent pendulum-wave equations determine the evolution of radiation field of steady-state PRL in plasma ripple. Eq.(8)-(9) are valid in weak or strong radiation fields, and for an arbitrary electron distribution. In the weak-field (or small signal) regime, these nonlinear equations may be linearized and the reference to the individual electron phase can be explicitly removed. In this case, the solution for the radiation amplitude a_n of the basic equations (8)-(9) may be expressed by supperposition of three terms as[12]

$$a_n = a_n(0)\sum_{j=1}^{3} C_{nj}\exp(i\lambda_{nj}\overline{z}_n) \tag{10}$$

where $a_n(0)$ is the initial field amplitude at the plasma-ripple entrance ($z = 0$), $C_{nj} = \lambda_{nj}/(3\lambda_{nj} - 2\nu_n)$ and λ_{nj} are the complex roots of the cubic equation

$$\lambda_n^3 - \nu_n\lambda_n^2 + 1 = 0 \tag{11}$$

which is well-known since the theory of travelling-wave tubes. Here the detuning parameter ν_n is approximately given by $\nu_n = -(\omega - \omega_n^+)/(\omega - \omega_n^-)/2\omega^2\rho_n$. Thus, the unstable spectrum can be divided into three parameter regimes. For $\nu_n^3 > 27/4$, there are three stable roots. As ν_n is decreasing [for $\nu_n < (27/4)^{1/3}$], the growth rate rapidly increases and peaks at ν_n=0. Then the growth rate slowly decreases with decreasing ν_n.

In the high-gain regime, evolution of the radiation field is dominated by the growth mode and the gain of the nth harmonic emission in PRL may be expressed by

$$G_n = \frac{1}{C_n^2}\exp(2\mid \mathrm{Im}(\lambda_n)\mid \tau_n) \tag{12}$$

As there are two resonant frequencies of the nth harmonic radiation, there exist two maxima of the gain curve for the nth harmonic. It should be noted that the two maxima of the gain curve move toward one another, until they give rise to a broad curve, as the plasma parameter increases toward the zero-slip condition $X_n=1$. When $X_n \sim 1$, the gain bandwidth is approximately given by $\Delta\omega/\omega \approx 2^{4/3}(nF_n^2\rho_0^3)^{1/6}$ [12]. Therefore, we may conjecture that the two resonant frequencies of PRL are well separated as $\kappa_n - 1 \gg (nF_n^2\rho_0^3)^{1/6}$. In the limit of $\kappa_n - 1 \gg (nF_n^2\rho_0^3)^{1/6}$ the maximum growth rate (the imaginary part of λ_n) occurs exactly on resonance, ν_n, and its value is $|\,\mathrm{Im}(\lambda_n)\,| = \sqrt{3}/2$.

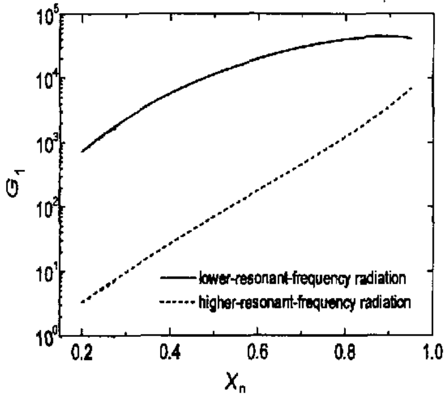

Fig.1 PRL gain G_1 of the fundamental emission at lower-resonant frequency (solid line) and higher-resonant frequency (dashed line) is plotted as a function of the parameter X_n, with $\alpha_R=0.3$, $\rho_0=0.02$, and $z=25/(k_R \cdot \rho_0)$

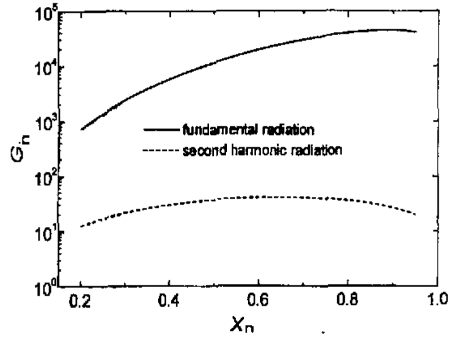

Fig.2 Gain G_n of the nth harmonic emission at the lower-resonant frequency in the PRL is shown as a function of the parameter X_n, for $n=1$ (solid line), and $n=2$ (dashed line), where $\alpha_R=0.3$, $\rho_0=0.02$, $z = 25/(k_R \cdot \rho_0)$

Obviously, the gain of the PRL is greatly dependent upon the parameter X_n. Fig.1 and Fig.2 provide illustrations. In Fig.1, we plot the gain G_1 of the fundamental emission as a function of the parameter X_n, where $a_R = 0.3$, $\rho_0=0.02$, $z = 25/(k_R \cdot \rho_0)$. It shows that in the high gain regime the gain itself is always positive and the gain of the lower-resonant-frequency emission is much higher than the gain of the higher-resonant-frequency emission. Therefore, one may conjecture that it is advantageous to operate the PRL at the lower-resonant frequencies. In Fig.2, the value of G_n for the nth harmonic emission at the lower-resonant frequencies in the PRL is plotted as a function of the parameter X_n for $n=1$ (solid line) and $n=2$ (dashed line), with $\alpha_R = 0.3$, $\rho_0 = 0.02$, $z = 25/(k_R \cdot \rho_0)$. From Fig.2, it is seen that the gain of the higher harmonic emission is much lower than the gain of the fundamental emission in the PRL. This is resulted from that the coupling between the REB and the fundamental ($n=1$) or the lower harmonic radiation field is much stronger than the coupling between the REB and the higher harmonic radiation field, since the coupling function $F_n = J_{-n}(n\sigma_{1,2})$ decreases with n for $\sigma_{1,2} < 1$. Then the PRL is expected to operate at the fundamental lower-resonant frequency.

Like the FEL, the PRL may be operated as an amplifier by injection of a small signal at the beginning of plasma ripple, or by Self-Amplified Spontaneous Emission (SASE)[13]. The saturation length of the SASE is $L_{\rm sat} = 4\pi L_{\rm Gn}$. By injecting a small signal at the beginning of plasma ripple, the plasma ripple length required to reach saturation will be significantly reduced. Fig.3 provides an illustration, in which the values of normalized intensity $|a_1|^2$ for the fundamental emission at the lower-resonant frequency in PRL is shown as a function of the dimensionless position $\bar{z}$ for SASE operation starting from a noise (solid line) and amplifier with an input signal $|a_1(0)|=0.01$ (dashed line).

From Fig.3, one knows that an initial exponential growth is followed by saturation and synchrotron oscillations. The nonlinear saturation of the transition-radiation emission in the PRL is due to the relativistic electron trapped by the ponderomotive potential well. The saturation efficiency caused by the phase rapping of the relativistic electrons in the ponderomotive potential well can be estimated from the relationship[14]

$$\eta_{\rm n} = (\gamma_{\rm z} - \gamma_{\rm n.ph})/(\gamma_{\rm z} - 1) \tag{13}$$

where $\gamma_{\rm n,ph} = (1 - \nu_{\rm n,ph}^2/c^2)^{-1/2}$, with $\nu_{\rm n,ph} \cong \beta_z c - 2{\rm Re}(\lambda_{\rm n})c/(k_{\rm n} + nk_R)L_{\rm Gn}$, the phase velocity of the ponderomotive potential with respect to the nth harmonic emission. In the limit of $\gamma \gg 1$ the efficiency of the nth harmonic emission may be approximately expressed by

$$\eta_{\rm n} \cong 2\rho_{\rm n} \mid {\rm Re}(\lambda_{\rm n}) \mid /n = \rho_{\rm n}/n \tag{14}$$

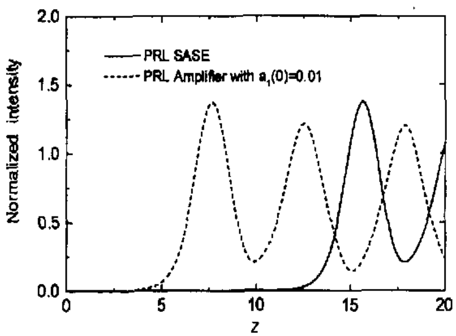

Fig.3 Values of normalized intensity $|a_1|^2$ for the fundamental emission at the lower-resonant frequency in PRL is shown as a function of the dimensionless position $\bar{z}$, for SASE operation starting from the noise (solid line) and the amplifier with input signals (dashed line)

Note that these expressions for the gain and the efficiency of the PRL have been based on the cold beam limit. The valid of the approximation would require $\nu_{\rm s}' \leq | \nu_{\rm n,ph}'$, where $\nu_{\rm s}'$ is the velocity spread and the superscript denotes the quantities in the beam frame. In the laboratory frame, for $\gamma \gg 1$, the condition becomes $\Delta\gamma/\gamma \leq \rho_{\rm n}$, where $\Delta\gamma$ is the energy spread.

These scaling laws have been applied to four numerical examples, whose parameters are given in Table 1. In these examples, it has been assumed that the PRL is operated at the fundamental lower-resonant frequency, the plasma-ripple modulation factor $\alpha_R=0.3$, and $X_1=0.95$. The results have been checked with a many-particle simulation based on Eqs. (8) and (9).

Table 1 Examples of short-pulse plasma-ripple transition-radiation laser scalings

	Microwave	Far-infrared	Mid-infrared	Near-infrared
E_b/MeV	1	4	10	27
I_b/A	100	500	1000	5000
n_b/cm^3	9.7×10^{11}	6.0×10^{13}	6.2×10^{14}	1.38×10^{15}
n_p/cm^3	9.7×10^{12}	6.0×10^{14}	6.2×10^{16}	1.38×10^{18}
λ_R/mm^{-1}	30	10	2	1.0
$\gamma\theta$	0.3	0.5	0.8	1.0
$\lambda/\mu m^{-1}$	4717	205	9.9	0.885
ρ_1	2.1×10^{-2}	2.0×10^{-2}	8.6×10^{-3}	3.1×10^{-3}
η_1	3.0×10^{-2}	2.9×10^{-2}	1.2×10^{-2}	4.4×10^{-3}
P_{out}/MW	3.0	58	120	590
L_{G1}/mm	280	100	46	66

4 Conclusion

In conclusion, we have studied the stimulated transition-radiation emission from a underdense REB passing through a dense plasma ripple. A set of self-consistent equations is developed to describe the evolution of the nth harmonic in this device. Analytic calculations of the small-signal gain and numerical computations of the nonlinear saturation of this emission are presented. Our analysis shows that it is advantageous to operate the PRL at the fundamental lower-resonant frequency. Moreover, some numerical examples of the PRL are given. It shows that this device may provide a coherent source ranging from the microwave to the near-infrared region. One of the main advantages of the PRL prior to the conventional FEL is the short plasma ripple period achievable in plasma. Compared to the conventional FEL, the PRL requires a lower REB energy and achieved higher efficiency to reach a given wavelength radiation. Compared to the conventional transition-radiation laser, the PRL becomes attractive because the problem of electron scattering in the dielectric is greatly diminished.

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