Criterion for reducible hydrodynamic equations of baryon–rich quark–gluon plasma

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Abstract Besides the state equation there exists another cubic algebraic equation about μ_f in the form of $\mu_f^3 + p\mu_f + q = 0$, which relates parameters, temperature T, chemical potential μ_f, and net quark number $n_{f\bar{f}}$ (flavor f) for a baryon-rich quark-gluon plasma (QGP). A criterion may be acquired simply according to Cardan formula of the solution of the above equation, which gives naturally a condition: if $n \ll 2\pi T^3/3\sqrt{3}$, one may approximately use the conservation of specific entropy, and then the set of hydrodynamic equation of baryon-rich QGP may be reduced to the set of hydrodynamic equation for baryon-free QGP.

Keywords Space-time evolution, Baryon-rich quark-gluon plasma, Cardan formula

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1 Introduction

The theoretical and experimental studies[1–3] reveal that as a good approximation a "full stopping scenario" [4] may be used to describe the dynamical evolution of a kind of new matter, the so-called the baryon-rich quark-gluon plasma (QGP)[5], that is different from the baryon-free QGP, in which two colliding heavy-ions are transparent for each other when colliding. Of course, in a baryon-rich QGP the net quark number density $n_{f\bar{f}}(\mathbf{r},t)$ and the baryon chemical potential $\mu_f(\mathbf{r},t)$ are not zero (where f labeling quark flavors), the variables such as temperature T, entropy S, net quark number $n_{f\bar{f}}$ (flavor f) and the baryon chemical potential μ_f are not all independent. However, one may find that some calculated results of dilepton invariant mass spectra for the baryon-rich QGP obtained by He[6] are similar to those obtained at lower initial temperature case for the baryon-free QGP[7,8], for instance, comparing Fig.5 in Ref.[8] and Fig.2 in Ref.[7] with the curves 1, 2, and 3 in Fig.2 in Ref.[6]. Notice that in Fig.2 in Ref.[6] $\mu_{b0}/\mu_0 = 1$ to 3 corresponding to $n_{b0}/n_N = 0.83$, 1.79 and 2.96, where n_{b0}, n_N and μ_{b0} are in turn the initial baryon density, normal nuclear density and initial baryon chemical potential. Hence it is quite interesting to ask under which condition the set of hydrodynamic equation for describing the baryon-rich QGP would reduce to the set of hydrodynamic equation used to describe baryon-free QGP except a trivial answer of $\mu_f(\mathbf{r},t)=0$ and $n(\mathbf{r},t)=0$. In this

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paper the author is going to give a significant and useful non-trivial criterion to answer the above question.

2 Formalism

For the convenience of discussion, let us recall some main equations suitable for baryon-rich QGP. Based upon the local thermodynamic equilibrium, the space-time evolution of a baryon-rich QGP is also governed by the conservation laws of the energy-momentum and baryon number: $\partial_\mu T^{\mu\nu}=0$ and $\partial_\mu B^\mu = 0$, with $T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu + pg^{\mu\nu}, B^\mu = n_b u^\mu$, where ϵ is the energy density, p the pressure, $g^{\mu\nu}$ the metric tensor (with g^{00}=-1), $u^\mu = \gamma(1,\mathbf{v})$ the four-velocity, where taking the velocity of light c=1 and $\mathbf{v}$ is the local flow velocity, and $\gamma = (1-\mathbf{v}^2)^{-1/2}$. The conservation law of baryons can be expressed equivalently as the net number $n_{f\bar{f}}(=n_f - n_{\bar{f}})$ conservation of each flavor f quarks separately. Koch *et al.* pointed that the net number of quarks with flavor f is given as follows.[9]

$$n_{ff} = g_f \int \frac{d^3\mathbf{k}}{(2\pi)^3}[(\exp(\sqrt{\mathbf{k}^2+m_f^2}-\mu_f)/T+1)^{-1} - (\exp(\sqrt{\mathbf{k}^2+m_f^2}+\mu_f)/T+1)^{-1}] \quad (1)$$

Eq.(1), in general, cannot be integrated as an analytical form. For the light quarks f = u, d, and for antiquarks $\bar{f} = \bar{u},\bar{d}$, g_f is determined by spin and color ($= 2_s \cdot 3_c = 6$). It must be kept in mind that considering the partons in QGP are deconfined and the chiral symmetry is approximately restored, $m_q \approx 0$, making use of $\mu_f = -\mu_{\bar{f}}$, one may obtain following relation:

$$n_{f\bar{f}} = \mu_f T^2 + (\frac{1}{\pi^2})\mu_f^3 \quad (2)$$

If the QGP arises from symmetric nuclear matter, the number of up- and down-quarks must be equal, so that with the help of a new set of thermodynamic relations derived for the baryon-rich QGP below: $d\epsilon = Tds + \mu_f dn$, $dp = sdT + nd\mu_f$, $w = \epsilon + p = Ts + \mu_f n$, where s is the entropy density, w the enthalpy density and n denotes n_{ff} here and v.i. one may obtain another equivalent form of the net number conservation law of quarks of flavor f: $\partial_\mu(nu^\mu) = 0$. Furthermore, by means of the above relations one may obtain the law of entropy density current conservation: $\partial_\mu(su^\mu) = 0$. Then in a 'full stopping scenario' the QGP will expand in a 3+1 spherical framework, the related equations mentioned above come to be

$$\frac{\partial}{\partial t}(n\cosh\eta) + \frac{1}{r^2}\frac{\partial}{\partial r}(nr^2\sinh\eta) = 0$$

$$\frac{\partial}{\partial t}(s\cosh\eta) + \frac{1}{r^2}\frac{\partial}{\partial r}(sr^2\sinh\eta) = 0$$

$$s[\frac{\partial}{\partial t}(T\sinh\eta) + \frac{\partial}{\partial r}(T\cosh\eta)] + n[\frac{\partial}{\partial t}(\mu_f\sinh\eta) + \frac{\partial}{\partial r}(\mu_f\cosh\eta)] = 0 \quad (3)$$

where $\eta = \tanh^{-1}v_r$.

Eqs.(3) are the concrete set of generalized relativistic hydrodynamic equations suitable for the baryon-rich QGP based upon three main hypotheses: the assumptions of the local thermodynamic equilibrium and adiabatic expansion as well as chiral symmetry is approximately restored ($m_q \approx 0$). Rearranging Eq.(2), one may obtain a cubic algebraic equation about μ_f

$$\mu^3 + \pi^2 T^2 \mu - \pi^2 n = 0 \tag{4}$$

According to Cardan formula of the solution of the equation $x^3 + px + q = 0$, a real root should be

$$x = \sqrt[3]{-\frac{q}{2} + \sqrt{(\frac{q}{2})^2 + (\frac{p}{3})^3}} + \sqrt[3]{-\frac{q}{2} - \sqrt{(\frac{q}{2})^2 + (\frac{p}{3})^3}} \tag{5}$$

if and only if the discriminant $\Delta = (\frac{q}{2})^2 + (\frac{p}{3})^3 \geq 0$. For Eq.(4), it always has $\Delta = \frac{\pi^4 n^2}{4} + \frac{\pi^6 T^6}{27} \gg 0$, therefore, the real root must be

$$\mu_f = \sqrt[3]{\frac{\pi^2 n}{2} + \sqrt{\Delta}} + \sqrt[3]{\frac{\pi^2 n}{2} - \sqrt{\Delta}}. \tag{6}$$

Then one may find that if $n \ll 2\pi T^3/3\sqrt{3}$, from Eq.(4) there must exist an approximate linear relation: $\mu_f/T \approx$ const., i. e. $\mu_b/T \approx$const.. Obviously for ultra-relativistic heavy-ion collisions with a finite baryon desity, thereupon, we have one more useful relation $n/T^3 \approx k_1$(a very small constant) also.

3 Discussion

At first sight, both of the conservation equations of the net number density of baryons and the entropy density current are of the same type partial differential equations, so that one may be able to guess that $s/n_b \approx$const, which is a reasonable approximation. However, people are not sure that (a) whether the local thermal equilibrium is really reached, slowly, or very fast or not at all? (b) whether another physical condition is necessary for the specific entropy conservation? (c) whether the chiral symmetry is approximately restored in an ultra-relativistic heavy-ion collision?

(1) One has to confirm question (a) in the further experiments. Since Kajantie *et al.*[10] the radiation from the central reaction region has been regarded as similar to the black-body radiation. If the surface temperature of the central region was as high as the mass of pion, $m_\pi \sim 140\,\text{MeV}$, for the time scales of collision process of the order of 10^{-23} sec, for instance, in Pb + Pb collision the total radiative dissipation across the boundary of the central region would be more than 200 GeV. Obviously, the ultra-relativistic heavy-ion collision is an irreversible dissipative process. Then, the first equation of Eqs.(3) should be a non-homogeneous equation. One hundred and fifty years ago, Clausius[11] thought that the calculation of entropy "does not complete the business" and "the discussion must be also extended to the radiant heat". Unfortunately, a generalized Gibbs' equation for the entropy including the radiative dissipation across the boundary has not been worked out yet so far. A new scheme given by the present

author[12] may be probably applied to solve this issue. We shall present a detail discussion of this topic elsewhere.

(2) Using approximate relations mentioned above, it turns out that $s \propto T^3$, then $s/n_b \approx$constant, that is the conservation of the specific entropy, which has been used in Ref.[13] without indicating the necessary approximate condition. Now we know that if one wants to use the conservation of specific entropy, besides the three main hypotheses used in Eqs.(3), the condition– $n \ll 2\pi T^3/3\sqrt{3}$ is necessary, under which Eqs.(3) may be reduced to the set of corresponding relativistic hydrodynamic equations suitable for the baryon-free QGP[8].

(3)It is accepted that in the case of broken chiral symmetry the quarks acquire a large effective mass ($m_u \approx m_d \approx 300\,\mathrm{MeV}$, $m_s \approx 500\,\mathrm{MeV}$) via interactions between themselves and the surrounding physical vacuum. Therefore, a better approximation ($m_f \neq 0$) leads to

$$n \approx k_2[\mu_f T^2 + (\frac{1}{\pi^2})(\mu_f^3 - m_f^3)] \tag{7}$$

where k_2 is the constant near one. A cubic algebraic equation about μ_f follows,

$$\mu_f^3 + T^2\pi^2\mu_f - (\frac{n}{k}\pi^2 + m_f^3) = 0 \tag{8}$$

One may obtain an approximate expression about μ_f,

$$\mu_f = aT + b(n, m_f^3)\frac{1}{T^2} \tag{9}$$

where both a and $b(n,\, m_f^3)$ are constant. Hence, it is explicit that in the case of broken chiral symmetry even the temperature may be taken as the unique parameter to describe the space-time evolution of the system, and the set of relativistic hydrodynamic equations for baryon-rich QGP may hardly be reduced to the set of corresponding relativistic hydrodynamic equations suitable for the baryon-free QGP.

References

1 Busza W. Nucl Phys, 1984, **A418**:635c
2 Bolz J, Ornik U, Weiner R M. Phys Rev, 1992, **C46**:2047
3 Barrette J. Phys Lett, 1994, **B333**:33
4 Åkesson T *et al.* Z Phys, 1988, **C38**:383
5 Danos M, Rafelski J. Baryon-rich quark-gluon plasma in nuclear collisions UCT-TPT/84
6 He Z J, Zhang J J. J Phys, 1995, **G21**:L49
7 Ko C M, Xia L H. Phys Rev Lett, 1989, **62**:1595
8 He Z J, Zhang J J, Li P L. Nucl Phys, 1991 **A532**:743
9 Koch P, Müller B, Rafelski J. Phys Rep, 1986, **142**:167
10 Kajantie K, Miettinen H. Z Phys, 1981, **C9**:341
11 Clausius R. A source book in physics, Magie William F ed. 1935, 235.
12 Ai H B. in: Proceedings of the Fourth Pacific Physics Conference, Ahn S H *et al* Ed. Seoul, August 13-17, 1990 (Singapore: World Scientific, 1991) Vol.1, 742~745
13 Kämpfer B *et al.* Z Phys, 1995, **A353**:71